L^p - L^q FAZOLAR ORASIDA HARDI TENGSIZLIGI

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Annotatsiya: Ushbu maqolada integral Hardy tengsizligi ta‘rifi, vaznli Hardy tengsizligi uchun olingan bir qancha muhim natijalar keltirilgan. Hardy operatori normasi ta‘riflangan .

Kalit so‘zlar: Hardy operatori normasi, Hardy tengsizligi, Integral Hardy tengsizligi, Hardy operatori normasining baholari.

Hardi-Littlevud 1930 yilda $q > p > 1\, va\, \gamma = \frac{1}{-p}$ uchun quyidagi tengsizlik o’rinli ekanligini isbotladi

$$(1)\quad \left(\int_0^\infty \left(\frac{1}{x}\int_0^x f(t)dt\right)^q x^{-1-q\gamma}dx\right)^{1/q} \le C\left(\int_0^\infty f(x)^p x^{-1-p\gamma}dx\right)^{1/p}.$$

(1) Tengsizlik 1958 yilda Flett tomonidan isbotsiz bayon qilingan va bir yildan so'ng u (1) tengsizlik $q \ge p \ge 1\, va\, \gamma \ne -1$ uchun to'g'ri deya nashr qildi. Umumiy vaznlilar $p \le q$ bilan u, v uchun natijalar 1971 yilda Uolshning maqolasida boshlangan. Shundan xulosa qilishimiz mumkinki, agar $1 \le p \le q \le \infty$ va $1/r = 1 - 1/p + 1/q$ bo‘lsa,

$$1 \le \frac{\|V\|_{L^p(v)\to L^q(u)}}{A_{p',q}} \le (p')^{1/p'} q^{1/q} r^{-1/r},$$

bo’ladi, bu yerda V quyidagicha aniqlanadi $Vf(x) := \int_0^x f(t)v(t)dt$ va

$$A_{p',q} = \sup_{x>0}\left(\int_x^\infty u(t)dt\right)^{1/p'}\left(\int_0^x v(t)dt\right)^{\frac{1}{q}}.$$

Bundan tashqari, Boyd-Erdosning 1972 yildagi qo'lyozmasida mualliflar $p = q$ uchun ularning isbotini $p < q$ gacha kengaytirish mumkinligini ta'kidlaganlar. Riemenschneidrga $1 < p \le q < \infty$ uchun $L^p(0,1)\, dan\, L^q(0,1)$ ga

$H_{u,v}$ operatorining ixchamligini isbotlashda uning chegaralanganligi kerak bo'ldi, u quyidagi funksiyaning chegaralanganligiga ekvivalent deb ta'riflagan

$$(0,1)da \quad h(x) \coloneqq \left(\int_x^1 |u(t)|^q dt\right)^{\frac{1}{q}} \left(\int_0^x |v(t)|^{p'} dt\right)^{1/p'}$$

$1 \leq p \leq q < \infty$ holatidagi muammoning to'liq yechilishi Mukkenxupt tipidagi oddiy isbot bilan birgalikda 1978 yilda Bredli maqolasida berilgan. U ushbu

$A \coloneqq \sup_{r>0}\left(\int_r^\infty u(x)dx\right)^{\frac{1}{q}}\left(\int_0^r v(x)^{1-p'}dx\right)^{1/p'} < \infty$ shart agar $1 \leq p, q < \infty$ bo'lsa, (2) $\left(\int_0^\infty\left(\int_0^x f(t)dt\right)^q u(x)dx\right)^{1/q} \leq C\left(\int_0^\infty f^p(x)v(x)dx\right)^{1/p}$ da zaruriyligini, bundan tashqari $1 \leq p \leq q < \infty$ bo'lganda yetarliligini ko'rsatgan. (2)dagi doimiy C uchun quyidagi baholar olingan

$$\begin{cases} A \leq C \leq Ap^{1/q}(p')^{1/p'}, agar\ 1 < p \leq q < \infty \\ C = A, \qquad agar\ p = 1. \end{cases},$$

.Quyidagi teoremada $p \neq q$ hol uchun vaznli Hardi tengsizligida bir nechta muhim natijalarni keltiramiz. Faraz qilaylik, $q < p$ bo'lganda $-\infty \leq a < b \leq \infty$ va $0 < q < \infty, 1 \leq p < \infty, \frac{1}{r} = \frac{1}{q} - \frac{1}{p}$ bo'lsin.

Teorema . (i) Agar $1 \leq p \leq q < \infty$ bo'lsa, u holda

$$(2) \quad \left(\int_a^b \left(\int_a^x f(t)dt\right)^q u(x)dx\right)^{1/q} \leq C\left(\int_a^b f(x)^p u(x)dx\right)^{1/p}$$

tengsizlik (a, b) da aniqlangan barcha o'lchanadigan $f(x) \geq 0$ funksiyalar uchun o'rinli bo'ladi faqat va faqat

$A \coloneqq \sup_{r\in(a,b)} \left(\int_r^b u(x)dx\right)^{\frac{1}{q}} \left(\int_a^r v(x)^{1-p'}dx\right)^{1/p'} < \infty$ bo'lsa.

Bundan tashqari, (2) dagi eng yaxshi doimiy C $1 < p < \infty$ uchun $A \leq C \leq \min\left((p)^{1/q}(p')^{1/p'}, q^{1/q}(q')^{1/p'}\right) A$ ni va $p = 1$ uchun $C = A$ ni qanoatlantiradi.

(ii) Agar $1 \leq q < p < \infty$ bo'lsa, u holda (2) tengsizlik faqat va faqat quyidagi holda bajariladi

$$A_1 := \left(\int_a^b \left(\int_x^b u(t)dt \right)^{\frac{r}{q}} \left(\int_a^x v(t)^{1-p'}dt \right)^{r/q'} v(x)^{1-p'}dx \right)^{1/r} < \infty .$$

Bundan tashqari, $\left(\frac{p-q}{p}\right)^{1/q'} A_1 \leq C \leq (p')^{1/pq'} q^{1/q} A_1$.

(iii) Agar $0 < q < 1 < p < \infty$ bo'lsa, u holda (2) tengsizlik faqat va faqat quyidagi holda bajariladi

$$A_2 := \left(\int_a^b \left(\int_x^b u(t)dt \right)^{\frac{r}{p}} \left(\int_a^x v(t)^{1-p'}dt \right)^{r/p'} u(x)dx \right)^{1/r} < \infty .$$

Bundan tashqari, $(q)^{1/p} A_2 \leq C \leq (p')^{1/r} q^{1/p} A_2$.

(iv) Agar $0 < q < 1 = p$ bo'lsa, u holda (2) tengsizlik faqat va faqat quyidagi holda bajariladi

$$A_3 := \left(\int_a^b \left(\int_x^b u(t)dt \, \underline{v}(x)^{-1} \right)^{\frac{q}{1-q}} u(x)dx \right)^{1/q-1} < \infty ,$$

Bunda $\underline{v}(x) = ess\ inf_{a<t<x}\, v(t)$. Bundan tashqari,

$$q(1-q)A_3 \leq C \leq (1-q)^{1-1/q} A_3.$$

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