LEBEG INTEGRALI VA LEBEG INTEGRALINI HISOBLASHDA UNING XOSSALARIDAN FOYDALANISH METODLARI

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Anotatsiya: Maqolada musbat Lebeg integrali va Lebeg integralini hisoblashda uning xossalaridan foydalanish usullari misollar orqali keltirilgan.

Kalit so‘zlar: Lebeg o’lchovi, Lebeg integrali, additivlik xossasi, chegaralangan va o’lchovli funksiya.

Fransuz matematigi, Parij universiteti professori, haqiqiy o‘zgaruvchining funksiyalarining zamonaviy nazariyasi asoschilaridan biri A.L.Lebeg Parij Fanlar Akademiyasi, London Qirollik jamiyati va boshqa ko‘plab ilmiy tashkilotlar a’zosi 1875-yil 28-iyunda tavallud topgan. Lebegning birinchi maqolalari, asosan, differensial geometriya va matematik analiz muammolariga tegishli edi. O'lchovlar nazariyasi va Lebeg integralining asosiy tushunchalari birinchi marta u tomonidan 1901 yilda "Aniq integralni umumlashtirish to'g'risida" maqolasida bayon etilgan. Quyida biz Lebeg integrali va uning xossalaridan foydalanishga to‘xtalamiz.

(Ω, Σ, μ) o‘lchovli fazo bo‘lsin. $\mathrm{E} \subset \Omega$ chekli o’lchovli to’plam, bu to’plamda aniqlangan $f(x)$ o’lchovli funksiya uchun

$$A < f(x) < B$$

bo’lsin. $[A, B]$ oraliqni $A = y_0 < y_1 < y_2 < \cdots < y_n = B$ bilan bo’lamiz va har bir yarim segmentga

$$E_k = \{x \in E: y_k \leq f(x) < y_{k+1}\}, k = \overline{0, n-1}$$

to’plamlarni mos qo’yamiz. Lebegning quyi va yuqori yig’indilari deb ataluvchi

$$s = \sum_{k=0}^{n-1} y_k \mu(E_k)$$

$$S=\sum_{k=0}^{n-1} y_{k+1}\mu(E_k)$$

yig'indilarni qaraymiz. Agar $\lambda=\max(y_{k+1}-y_k)$ deb olsak, u holda

$$0\le S-s\le \lambda\mu(E).$$

tengsizlikda $\lambda\to 0$ bo'lganda $\{S\}$ va $\{s\}$ yig'indilar biror songa intiladi va bu son $f(x)$ funksiyaning E to'plam bo'yicha Lebeg integrali deyiladi. Lebeg integrali $(L)\int_E f(x)d\mu(x)$ kabi belgilanadi.

1-masala. $f(x)$ chegaralangan o'lchovli funksiya E o'lchovli to'plamda $m\le f(x)\le M$ tengsizlikni qanoatlantirsa, u holda

$$m\mu(E)\le (L)\int_E f(x)d\mu(x)\le M\mu(E)$$

o'rinlidir.

Lebeg integralining ushbu xossasidan foydalanib

$$\frac{2}{\sqrt[4]{e}}\le (L)\int_{[-1;1]} e^{x^2+x}d\mu\le 2\cdot e^2$$

tengsizlikni isbotlaymiz.

Bu yerda μ haqiqiy sonlar to'plamidagi Lebeg o'lchovi. Integral ostidagi $f(x)=e^{x^2+x}$ funksiyaning $f'(x)=(2x+1)e^{x^2+x}$ hosilasidan foydalanib, uning $\left[-1;-\frac{1}{2}\right]$ oraliqda kamayuvchi, $\left[-\frac{1}{2};1\right]$ oraliqda o'suvchi ekanligini aniqlanadi.

Demak, $m=\min\limits_{[-1;1]} f(x)=f\left(-\frac{1}{2}\right)=e^{\frac{1}{4}-\frac{1}{2}}=e^{-\frac{1}{4}}=\frac{1}{\sqrt[4]{e}}$

$$M=\min_{[-1;1]} f(x)=f(1)=e^2.$$

U holda $e^{-\frac{1}{4}}\mu[-1;1]\le (L)\int_{[-1;1]} e^{x^2+x}d\mu\le e^2\mu[-1;1]$, ya'ni

$$\frac{2}{\sqrt[4]{e}}\le (L)\int_{[-1;1]} e^{x^2+x}d\mu\le 2\cdot e^2.$$

2-masala. Agar $E,E_i\quad i=1,2,\dots,n,\dots$ o'lchovli to'plamlar bo'lib,

$E = \bigcup_{i=1}^{n} E_i$ (n-natural son) va $E_i \cap E_k = \emptyset, i \neq k$ va $f(x)$ funksiya E to'plamda chegaralangan va o'lchovli bo'lsa, u holda

$(L)\int_E f(x)d\mu = \sum_{i=1}^{n}(L)\int_{E_i} f(x)\,d\mu$ formuladan foydalanib

$$(L) \int_{[-3;2]} (-1)^{|x|} d\mu$$

integralni hisoblaymiz.

$(L)\int_{[-3;2]}(-1)^{|x|}d\mu = \int_{[-3;2]}(-1)^{-3}d\mu + + \int_{[-2;-1]}(-1)^{-2}d\mu +$
$(L)\int_{[-1;0]}(-1)^{-1}d\mu + (L)\int_{[0;1]} d\mu + \int_{[1;2]}(-1)d\mu =$
$= -1 - 1 - 1 + 1 - 1 = -1$

3-masala. Agar E, E_i $i = \overline{1,\infty}$ o'lchovli to'plamlar bo'lib, $E = \bigcup_{i=1}^{\infty} E_i$ $E_i \cap E_k = \emptyset$, $i \neq k$ va $f(x)$ funksiya E to'plamda chegaralangan va o'lchovli bo'lsa, u holda

$$(L)\int_E f(x)d\mu = \sum_{i=1}^{\infty}\int_{E_i} f(x)d\mu$$

Integralning bu xossasi uning to'la additivligi deyiladi.

Ushbu xossadan foydalanib, $f(t) = e^{-[t]}$ funksiyaning $(0,+\infty)$ oraliqdagi Lebeg integralini hisoblang. $n \leq t < n+1$ oraliqda $[t] = n$ bo'lganligidan, bu oraliqda $f(t) = e^{-n}$ bo'ladi.

$$(L)\int_{(0,+\infty)} f(t)dt = \sum_{n=0}^{+\infty}\int_n^{n+1} f(t)dt = \sum_{n=0}^{+\infty}\int_n^{n+1} e^{-n}dt = \sum_{n=0}^{+\infty} e^{-n} = \frac{e}{e-1}.$$

4-masala. $(0,\infty)$ oraliqda $f(t) = \frac{1}{[t+1][t+2]}$ funksiyaning Lebeg integralini hisoblaymiz.

Agar $n \leq t < n+1$ bo'lsa, u holda

$$f(t) = \frac{1}{(n+1)(n+2)}$$

bo'ladi. Integralning to'la additivlik xossasiga asosan

$$(L)\int\limits_{(0,\infty)} f(t)dt = \sum_{n=0}^{\infty}\int\limits_{n}^{n+1} f(t)dt = \sum_{n=0}^{\infty}\int\limits_{n}^{n+1}\frac{1}{(n+1)(n+2)}dt =$$

$$= \sum_{n=0}^{\infty}\frac{1}{(n+1)(n+2)} = \sum_{n=0}^{\infty}\left(\frac{1}{(n+1)} - \frac{1}{(n+2)}\right) = \left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \cdots$$

$$+\left(\frac{1}{n+1} - \frac{1}{n+2}\right) + \cdots = 1$$

5-masala. Agar $f(x)$ funksiya $[a,b]$ oraliqda Riman ma'nosida integrallanuvchi bo'lsa, u holda uning uzilish nuqtalari to'plami nol o'lchoviga ega.

Ushbu xossadan foydalanib quyidagi misolni yechamiz: $[0,1]$ segmentda o'lchovi $\frac{1}{2}$ ga teng bo'lib, hech qayerda zich bo'lmagan mukammal *E* to'plam tuzilgan; ushbu to'plamning to'ldiruvchi oraliqlari uzunliklarining kamayib borishi tartibida $(\propto_1, \beta_1), (\propto_2, \beta_2), \dots, (\propto_n, \beta_n), \dots$ kabi belgilangan. Keyin $[0,1]$ segmentda $f(x)$ funksiya quyidagicha aniqlangan:

$$f(x) = 0, agar\, x \in E,$$

$$f(x) = 1, agar\, x \in (\propto_n, \beta_n)$$

$f(x)$ funksiya $\left[\propto_n, \frac{\propto_n+\beta_n}{2}\right]$ va $\left[\frac{\propto_n+\beta_n}{2}, \beta_n\right]$ segmentlarda chiziqli.

Ushbu funksiya $[0,1]$ segmentda Riman ma'nosida integrallanuvchi bo'ladimi? Lebeg ma'nosidachi ? Uning $[0,1]$ segmenti bo'yicha Lebeg integrali nimaga teng ?

$f(x)$ funksiya uzilish nuqtalari to'plami *E* ning o'lchovi 0 dan katta bo;lgani uchun Riman ma'nosida integrallanuvchi emas, lekin Lebeg ma'nosida integrallanuvchi.

$f(x)$ funksiyaning $[0,1]$ segmentdagi Lebeg integralini hisoblaymiz:

$$(L)\int_{[0;1]} f(x)dx = (L)\int\limits_{E} f(x)dx + (L)\int\limits_{[0,1]/E} f(x)dx = \sum_{n=1}^{\infty}(L)\int\limits_{\propto_n}^{\beta_n} f(x)dx =$$

$$= \sum_{n=1}^{\infty} (R) \int_{\alpha_n}^{\beta_n} f(x)dx = \sum_{n=1}^{\infty} \frac{\beta_n - \alpha_n}{2} \cdot 1;$$

Demak, $(L) \int_{[0,1]} f(x)dx = \sum_{n=1}^{\infty} \frac{\beta_n - \alpha_n}{2} \cdot 1$

Lekin, $\sum_{n=1}^{\infty} (\beta_n - \alpha_n)$ bu to'ldiruvchi oraliqlar uzunliklarining yig'indisi; u $[0,1]/E$ to'plamning o'lchovi $\frac{1}{2}$ ga teng.

Demak, $(L) \int_{[0,1]} f(x)dx = \frac{1}{4}$.

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