TARTIBI 2 GA TENG BO'LGAN MATRITSALAR ALGEBRASIDA NOLNING CHAP VA O'NG BO'LUVCHILARINING MAVJUDLIGI

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Annotatsiya. Ushbu maqolada tartibi ikkiga teng bo'lgan matritsalar algebrasida nolning chap va o'ng bo'luvchilarining mavjudligi haqida so'z yuritilgan.

Kalit so'zlar. Matritsa, halqa, kommutativlik, kompleks son, kvadrat matritsa.

Elementlari biror R halqadan olingan n-tartibli kvadrat matritsalar to'plami M_n(R) matritsalarni qo'shish va ko'paytirish amallariga nisbatan halqa tashkil qiladi. Matritsalarni ko'paytirish amali uchun kommutativlik o'rinli bo'lmaganligi sababli (M_n(R), +,•) nokommutativ halqa bo'ladi.

Demak, M_n(Z), M_n(Q), M_n(R) va M_n(C) halqalardir, ya'ni elementlari mos ravishda butun, ratsional, haqiqiy, kompleks sonlardan iborat bo'lgan n-tartibli kvadrat matritsalar to'plami nokommutativ halqa bo'ladi.

Ta'rif: Halqaning noldan farqli a ϵ R elementi uchun, shunday noldan farqli b ϵ R topilib, a•b = 0 shart bajarilsa, a element nolning chap bo'luvchisi, b•a = 0 shart bajarilgan holda a element nolning o'ng bo'luvchisi deb ataladi.

Matritsalar halqasi ((M_2(R), +,•)) nokommutativ halqa bo'lishi bilan birgalikda, nolning bo'luvchilariga ega.

Misol. A = $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ matritsa uchun nolning chap va o'ng bo'luvchilarini topamiz. Ta'rifga ko'ra qandaydir B matritsa mavjud bo'lib A•B = 0 tenglik bajarilsa, B matritsa nolning chap bo'luvchisi bo'ladi. Faraz qilaylik B = $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ hamda (b_{11}, b_{12}, b_{21}, $b_{22} \epsilon R$) bo'lsin va A•B = 0 ga ko'ra

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \bullet \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{11} & b_{12} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$b_{11} = 0$, $b_{12} = 0$ bo'ladi. Endi biz B matritsani B = $\begin{pmatrix} 0 & 0 \\ b_{21} & b_{22} \end{pmatrix}$ ko'rinishida yoza olamiz va u nolning o'ng bo'luvchisi bo'ladi. $(b_{21}, b_{22} \epsilon R)$ ekanligidan $B^* = \begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$ $B^{**} = \begin{pmatrix} 0 & 0 \\ -3 & 4 \end{pmatrix}$ $B^{***} = \begin{pmatrix} 0 & 0 \\ 7 & 2 \end{pmatrix}$ lar barchasi nolning o'ng bo'luvchisi bo'ladi.

Endi biz A matritsamiz uchun nolning chap bo'luvchisini topamiz va buning uchun qandaydir C matritsa topilib C•A = 0 tenglik bajarilishi lozim. C = $\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$ $(c_{11}, c_{12}, c_{21}, c_{22} \epsilon R)$. Demak C•A = 0

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \bullet \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} c_{11} + c_{12} & 0 \\ c_{21} + c_{22} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$c_{11} = -c_{12}$ va $c_{21} = -c_{22}$, demak C matritsa satrlaridagi elementlari bir-biriga qarama-qarshi bo'lishi kerak ekan, ya'ni C = $\begin{pmatrix} c_{11} & -c_{11} \\ c_{21} & -c_{21} \end{pmatrix}$ bo'lsa, u holda C nolning chap bo'luvchisi bo'ladi. Xususiy holda $C^* = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}$ $C^{**} = \begin{pmatrix} 3 & -3 \\ 7 & -7 \end{pmatrix}$ kabi bo'lishi mumkin.

Teorema.Tartibi ikkiga teng bo'lgan, diogonal bo'lmagan matritsalar halqasi har doim nolning chap va o'ng bo'luvchilari ga ega.

Isbot. ((M_2(R), +,•)) da A = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ va B= $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ bo'lsin ta'rifga ko'ra A•B = 0 bo'lib A ≠ 0 va B ≠ 0 bo'lsa gina, A va B matritsalar mos ravishda nolning chap va o'ng bo'luvchilari bo'ladi, ya'ni

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \bullet \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

shart bajarilsa.

Matritsalarni ko'payritish qoidasiga ko'ra:

$$\begin{cases} a_{11} \bullet b_{11} + a_{12} \bullet b_{21} = 0 \\ a_{11} \bullet b_{12} + a_{12} \bullet b_{22} = 0 \\ a_{21} \bullet b_{11} + a_{22} \bullet b_{21} = 0 \\ a_{21} \bullet b_{12} + a_{22} \bullet b_{22} = 0 \end{cases} \qquad (1)$$

tengliklar o'rinli. Agar biz $a_{11}, a_{12}, \dots, b_{21}, b_{22}$ larni noma'lumlar deb qarab chiziqli tenglamalar sistemasining yechimini topsak, uning yechimi chaksiz ko'pdir, chunki noma'lumlar soni 8 ta, tenglamalar soni esa 4 ta. Bu berilgan chiziqli tenglamalar sistemasi cheksiz ko'p yechimga ega bo'ladi.

Eslatma. Diogonal matritsa hech qachon nolning chap va o'ng bo'luvchilariga ega emas!

A = $\begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$ bu yerda $a_{11} \neq 0$, $a_{22} \neq 0$ va B = $\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ bo'lganda

$$A \bullet B = \begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix} \bullet \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} \bullet b_{11} & a_{11} \bullet b_{12} \\ a_{22} \bullet b_{21} & a_{22} \bullet b_{22} \end{pmatrix}$$

ravshanki bu matritsa nolga teng emas. Xuddi shuningdek A va B matritsalarning o'rinlarini almashtirsak ham yoki A matritsada ikkinchi diogonal elementlari noldan farqli bo'lgan holatda ham yuqoridagi kabi natijaga ega bo'lamiz.

FOYDALANILGAN ADABIYOTLAR.

1. Sh.A.Ayupov, B.A.Omirov, A.X.Xudoyberdiyev Abstrakt algebra (o'quv qo'llanma)-Toshkent:2022 O'zRFA "Fan" nashriyoti (155-157)

2. Sh.A.Ayupov, B.A.Omirov, A.X.Xudoyberdiyev, F.H.Xaydarov Algebra va sonlar nazariyasi (o'quv qo'llanma) Toshkent – 2019 (80-82)