MAPLE DASTURIDA GEOMETRIK VA BINOMIL QONUN BO`YICHA TAQSIMLANGAN TASODIFIY MIQDORLARNING YUQORI TARTIBLI MOMENTLARINI O`RGANISH

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Anotatsiya. Ushbu ishda boshlang`ich va markaziy momentlar haqida muhim faktlar keltirilgan.Geometrik va binomial qonun bo`yicha taqsimlangan tasodifiy miqdorlarning yuqori tartibli boshlang`ich momentlari topilgan va grafiklari chizilgan.

**Kalit so`zlar.** Geometrik taqsimot, binomial taqsimot, boshlang`ich moment, markaziy moment.

Bu ishda biz geometrik va binomial qonun bo`yicha taqsimlangan tasodifiy miqdorlarning yuqori tartibli boshlang`ich va markaziy momentlarini o`rganamiz.

Ixtiyoriy taqsimot funksiya $F(x)$ ning hamma tartibdagi momentlari

$$m_1, m_2, ..., m_n, ...$$

mavjud bo`lsin. Bu momentlar $F(x)$ funksiyani bir qiymatli aniqlaydi degan masalani qo`yamiz. Bu masala matematik analizdagi "**momentlar problemasi**" deb ataladigan umumiy masala bilan bog`liq va uning yechimidan quyidagi natija kelib chiqadi. Agar

$$\sum_{n=1}^{\infty} \frac{m_n}{n!} r^n < \infty$$

qator biror $r > 0$ uchun yaqinlashsa, $F(x)$ funksiya $m_1, m_2, ..., m_n, ...$ momentlarga ega bo`lgan funksiya bo`ladi.

1-ta'rif X tasodifiy miqdorning $k-$tartibli boshlang`ich momenti deb, diskret tasodifiy miqdorlar uchun

$$a_k = MX^k = \sum_{-\infty}^{\infty} x_i^k P\{X = x_i\}$$

ifoda aytiladi.

2-ta'rif X tasodifiy miqdorning k – tartibli markaziy moment deb, diskret tasodifiy miqdorlar uchun

$$b_k = M(X - MX)^k = \sum_{-\infty}^{\infty} (x_i - MX)^k P\{X = x_i\}$$

ifoda aytiladi.

Endi geometrik va binomial qonun bo`yicha taqsimlangan tasodifiy miqdorlarning yuqori tartibli boshlang`ich va markaziy momentlarini Maple dasturida o`rganamiz .Yig`indini hisoblashda **summation** buyrug`idan foydalanamiz. Grafigini chizishda **plot** buyrug`i va uning parametrlaridan foydalanamiz.

Plot buyrug`i va uning parametrlari.** Bir o`zgaruvchili funksiyaning grafigini (Ox** o`qi bo`yicha intervalda **a<=x<=b** va **Oy** o`qi bo`yicha **c<=x<=d** intervalda) yasash uchun plot buyrug`i ishlatiladi.Uning umumiy ko`rinishi quyidagicha: **plot(f(x),x=a..b,y=c..d, parametr),** bu yerda **parameter-**tasvirni boshqarish parametrlari.Agar u ko`rsatilmasa jimlik bo`yicha o`rnatishdan foydalaniladi. Shu bilan birga tasvirlarga tuzatishlar kiritish vositalar paneli orqali ham amalga oshiriladi.

Geometrik taqsimot

Agar X tasodifiy miqdor $1, 2, ..., m, ...$ qiymatlarni

$$p_k = P\{X = k\} = q^{k-1} p$$

ehtimolliklar bilan qabul qilsa, u geometrik qonuni bo`yicha taqsimlangan tasodifiy miqdor deyiladi.Bu yerda $p = 1 - q \in (0,1)$.

Geometrik qonun bo`yicha taqsimlangan tasodifiy miqdorlarga misol sifatida quyidagilarni olish mumkin: sifatsiz mahsulot chiqqunga qadar tekshirilgan mahsulotlar soni; gerb tomoni tushgunga qadar tashlangan tangalar soni; nishonga tekkunga qadar otilgan o`qlar soni va hokazo.Endi geometrik qonun bo`yicha taqsimlangan tasodifiy miqdorning boshlang`ich momentlarini maple dasturida qaraylik.

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> X := p·(1 - p)^(k - 1)
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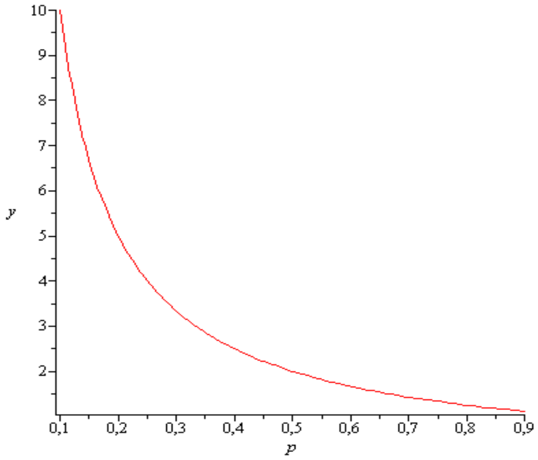
$$X := p\ (1 - p)^{k-1}$$

Bu taqsimotning birinchi tartibli boshlang`ich momentini uning matematik kutilmasiga teng va quyidagicha hisoblanadi.

> $a_1 := \sum_{k=1}^{\infty} k \cdot X$

$$a_1 := \frac{1}{p}$$

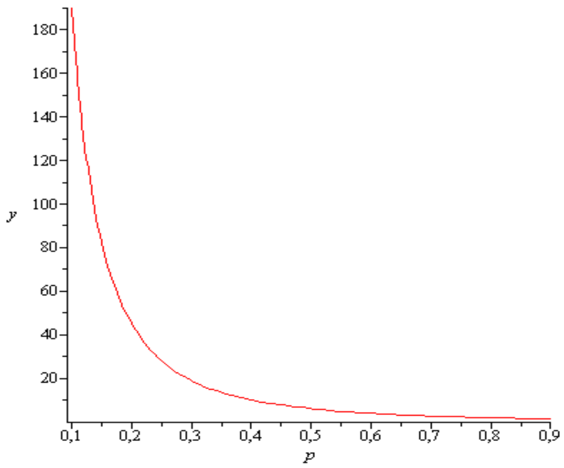
> $plot\left(\frac{1}{p}, p = 0.1\,..0.9, labels = [p, y], thickness\ = 1\right);$



> $a_2 := \sum_{k=1}^{\infty} k^2 \cdot X$

$$a_2 := \frac{-p + 2}{p^2}$$

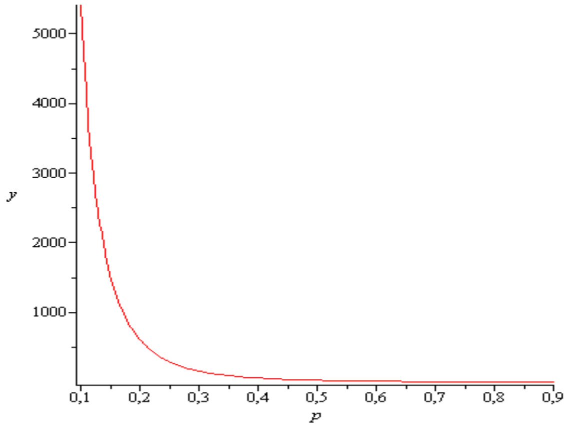
> $plot\left(\frac{-p + 2}{p^2}, p = 0.1\,..0.9, labels = [p, y], thickness\ = 1\right);$



> $a_3 := \sum_{k=1}^{\infty} k^3 \cdot X$

$$a_3 := \frac{p^2 - 6\,p + 6}{p^3}$$

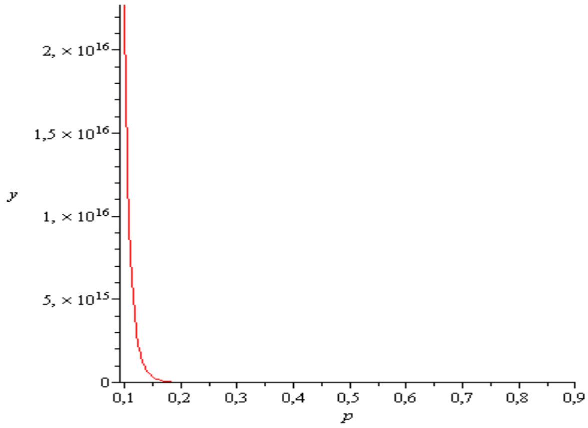
> $plot\left(\frac{p^2 - 6\,p + 6}{p^3}, p = 0.1\,..0.9, labels = [p, y], thickness\ = 1\right);$



> $a_{10} := \sum_{k=1}^{\infty} k^{10} \cdot X$

$$a_{10} := -\frac{1}{p^{10}} \left((p-2) \left(p^8 - 1020 p^7 + 53940 p^6 - 710640 p^5 + 3681720 p^4 - 9072000 p^3 + 11491200 p^2 - 7257600 p + 1814400 \right) \right)$$

> $plot\left(-\frac{1}{p^{10}} \left((p-2) \left(p^8 - 1020 p^7 + 53940 p^6 - 710640 p^5 + 3681720 p^4 - 9072000 p^3 + 11491200 p^2 - 7257600 p + 1814400 \right) \right), p = 0.1 ..0.9, labels = [p, y], thickness = 1 \right);$

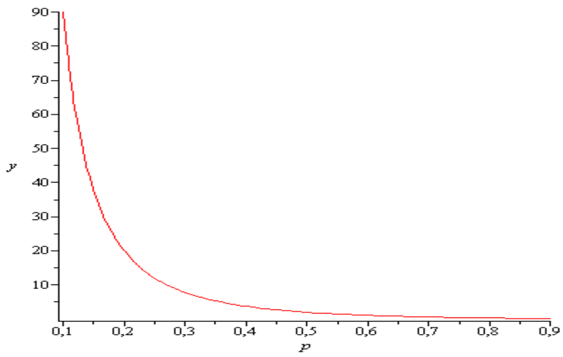


Bu taqsimotning boshqa tartibli boshlang`ich momentlarini ham shu tarzda o`rganiladi.Endi geometrik qonun bo`yicha taqsimlangan tasodifiy miqdorning yuqori tartibli markaziy momentlarini hisoblaymiz va grafigini chizamiz. 2-tartibli markaziy moment geometrik qonun bo`yicha taqsimlangan tasodifiy miqdorning dispersiyasini beradi.

> $b_2 := \sum_{k=1}^{\infty} (k - a_1)^2 \cdot X$

$$b_2 := \frac{1-p}{p^2}$$

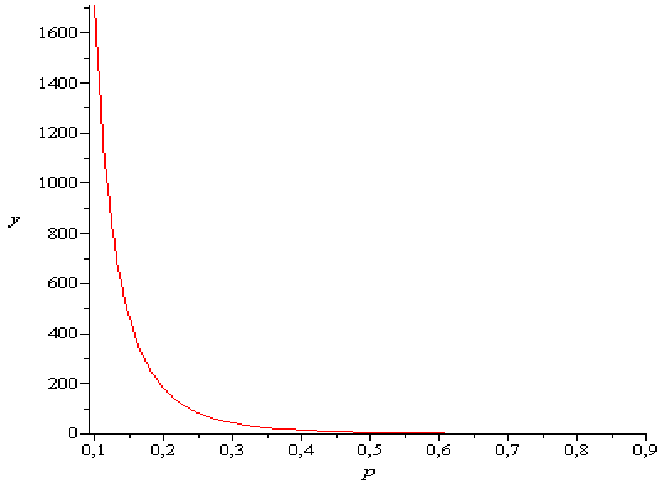
> $plot\left(\frac{1-p}{p^2}, p = 0.1 ..0.9, labels = [p, y], thickness = 1 \right);$



> $b_3 := \sum_{k=1}^{\infty} (k - a_1)^3 \cdot X$

$$b_3 := \frac{(-1+p)(p-2)}{p^3}$$

> $plot\left(\frac{(-1+p)(p-2)}{p^3}, p = 0.1..0.9, labels = [p, y], thickness = 1\right);$



Binomial taqsimot

Agar X tasodifiy miqdor $0,1,2,...,n$ qiymatlarni

$$p_k = P\{X = k\} = \frac{n!}{k!(n-k)!} p^k q^{n-k},$$

ehtimolliklar bilan qabul qilsa, u binomial qonun bo`yicha taqsimlangan tasodifiy miqdor deyiladi. Bu yerda $p = 1 - q \in (0,1)$. Endi binomial qonun bo`yicha taqsimlangan tasodifiy miqdorning boshlang`ich momentlarini maple dasturida qaraylik.

> $X := \frac{n!}{k!(n-k)!} p^k \cdot (1-p)^{n-k}$

$$X := \frac{n!\, p^k\, (1-p)^{n-k}}{k!\,(n-k)!}$$

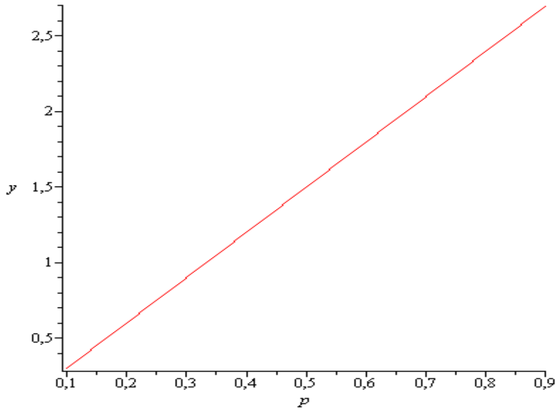
Quyida binomial qonun bo`yicha taqsimlangan tasodifiy miqdorning n=3 bo`lganda grafigini chizamiz .

Bu taqsimotning birinchi tartibli boshlang`ich moment uning matematik kutilmasiga teng va quyidagicha hisoblanadi.

> $a_1 := \sum_{k=0}^{n} k \cdot X$

$$a_1 := p\, n$$

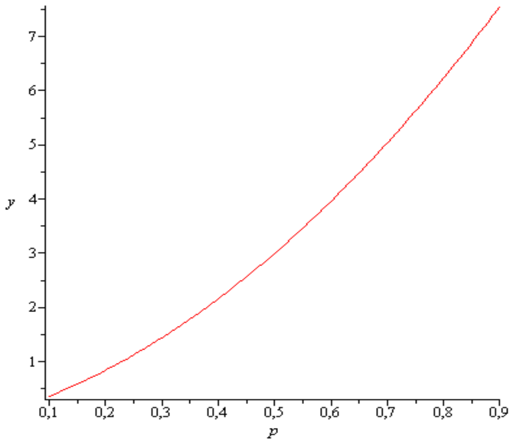
> $plot(3 \cdot p, p = 0.1 ..0.9, labels = [p, y], thickness = 1);$



> $a_2 := \sum_{k=0}^{n} k^2 \cdot X;$

$$a_2 := p\,(p\, n - p + 1)\, n$$

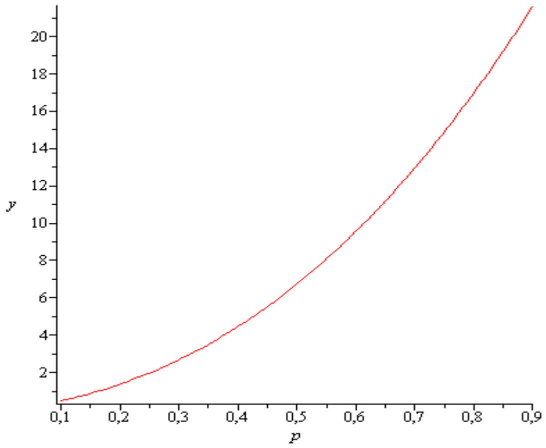
> $plot(p \cdot (p \cdot 3 - p + 1) \cdot 3, p = 0.1 ..0.9, labels = [p, y], thickness = 1);$



> $a_3 := \sum_{k=0}^{n} k^3 \cdot X;$

$$a_3 := p\, n\, \left(p^2 n^2 + 3\, p\, n - 3\, p^2\, n - 3\, p + 1 + 2\, p^2\right)$$

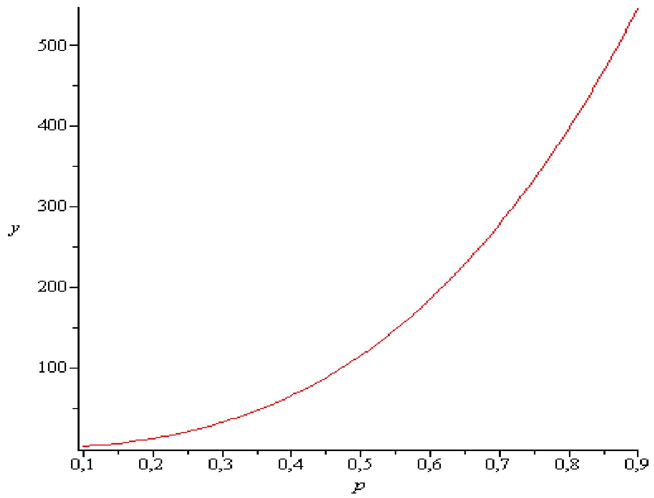
> $plot\left(p \cdot 3\, \left(p^2 \cdot 3^2 + 3 \cdot p \cdot 3 - 3 \cdot p^2 \cdot 3 - 3 \cdot p + 1 + 2 \cdot p^2\right), p = 0.1 ..0.9, labels = [p, y], thickness = 1\right);$



> $a_6 := \sum_{k=0}^{n} k^6 \cdot X$

$$a_6 := p\, n\, \left(p^5 n^5 + 15 p^4 n^4 - 15 p^5 n^4 + 85 p^5 n^3 + 65 p^3 n^3 - 150 p^4 n^3 - 225 p^5 n^2 - 390 p^3 n^2 + 525 p^4 n^2 + 90 p^2 n^2 + 274 p^5 n + 715 p^3 n - 750 p^4 n - 270 p^2 n + 31 p\, n + 1 - 390 p^3 - 120 p^5 - 31 p + 180 p^2 + 360 p^4\right)$$

> $plot\left(p \cdot 3\, \left(p^5 \cdot 3^5 + 15 \cdot p^4 \cdot 3^4 - 15 \cdot p^5 \cdot 3^4 + 85 \cdot p^5 \cdot 3^3 + 65 \cdot p^3 \cdot 3^3 - 150 \cdot p^4 \cdot 3^3 - 225 \cdot p^5 \cdot 3^2 - 390 \cdot p^3 \cdot 3^2 + 525 \cdot p^4 \cdot 3^2 + 90 \cdot p^2 \cdot 3^2 + 274 \cdot p^5 \cdot 3 + 715 \cdot p^3 \cdot 3 - 750 \cdot p^4 \cdot 3 - 270 \cdot p^2 \cdot 3 + 31 \cdot p \cdot 3 + 1 - 390 \cdot p^3 - 120 \cdot p^5 - 31 \cdot p + 180 \cdot p^2 + 360 \cdot p^4\right), p = 0.1\,..0.9, labels = [p, y], thickness = 1\right);$

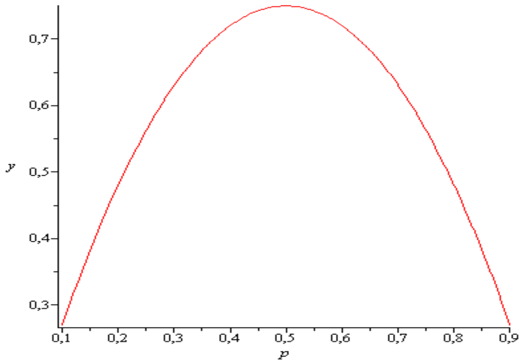


Bu taqsimotning boshqa tartibli boshlang`ich momentlarini ham shu tarzda o`rganiladi.Endi binomial qonun bo`yicha taqsimlangan tasodifiy miqdorning yuqori tartibli markaziy momentlarini hisoblaymiz va grafigini chizamiz. 2-tartibli markaziy moment binomial qonun bo`yicha taqsimlangan tasodifiy miqdorning dispersiyasini beradi.

> $b_2 = \sum_{k=0}^{n} (k - p\, n)^2 \cdot X;$

$$b_2 = \left(-p^2 + p\right) n$$

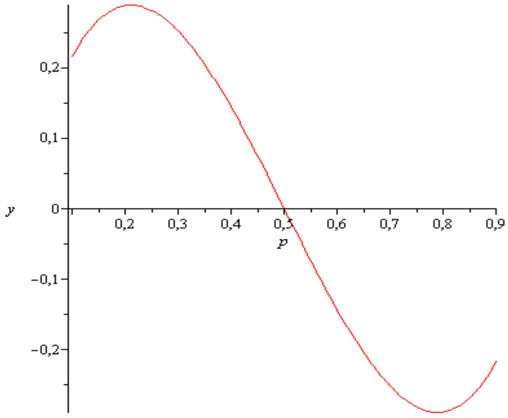
> $plot\left(\left(-p^2+p\right)\cdot 3, p = 0.1..0.9, labels = [p, y], thickness = 1\right);$



> $b_3 = \sum_{k=0}^{n} (k - p\,n)^3 \cdot X$

$$b_3 = \left(2\,p^3 + p - 3\,p^2\right) n$$

> $plot\left(\left(2\,p^3 + p - 3\,p^2\right)\cdot 3, p = 0.1..0.9, labels = [p, y], thickness = 1\right);$



Yuqorida keltirilagan tasodifiy miqdorlarning grafigi tanlab olingan $0{,}1 < p < 0{,}9$ ehtimolda momentlarning tartibi oshib borganda o`zgarishini ko`rishimiz mumkin. Bu bizga geometrik va binomial qonun bo`yicha taqsimlangan tasodifiy miqdorlarga oid misollarni yechishimizda muhim o`rin tutadi.

Adabiyotlar

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