BIR ZARRACHALI SHREDINGER OPERATORI XOS QIYMATI UCHUN ASSIMPTOTIK FORMULALAR

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ANNOTATSIYA

Ushbu ishda bir o'lchamli panjarada tashqi maydon ta'siri bilan kontakt ta'sirlashuvchi bir kvant zarracha harakatini tavsiflovchi bir zarrachali gamiltonianga mos bir zarrachali diskret Shredinger operatori h_μ *ning zarrachalar ta'sirlashuv energiyasi* $\mu > 0$ *ga bog'liq chapda yoki o'ngda yagona xos qiymatga ega ekanligi isbotlangan.*

Ushbu xos qiymat $z(\mu)$, $\mu > 1$ *ning aniq ko'rinishi topilgan. O'zaro ta'sir parametri* $\mu > 1$ *ga monoton va uzluksiz bog'liqligi ko'rsatilib, moduli yetarlicha kichik* $\mu > 1$ *larda ushbu xos qiymatlar uchun yaqinlashuvchi yoyilmalar olingan .*

Kalit so'zlar: *panjara, operator, xos qiymat, spektr, muhim spektr, yoyilma.*

ASSYMPTOTIC FORMULAS FOR EIGENVALUE OF ONE PARTICLE SCHRODINGER OPERATOR

ABSTRACT

In this study, one particle interaction energy of one particle discrete Schrödinger operator h_μ *corresponding to a quantum particle gamiltonian describing the motion of a quantum particle in contact with an external field effect in a one-dimensional grid depends on* $\mu > 0$ *left or right proved to have a eigenvalue.*

This eigenvalue $z(\mu)$, $\mu > 1$ *a clear view of was found. The monotonic and continuous dependence of the interaction parameter on* $\mu > 1$ *is shown, and approximate distributions are obtained for these eigenvalues at sufficiently small modulus* $\mu > 1$.

Keywords: *lattince**,** operator, eigenvalue, spectrum, essential spectrum.*

KIRISH

Tashqi maydon ta'siridagi bir kvant zarracha harakatini tavsiflovchi bir zarrachali gamiltonianlar hamda qisqa masofada ta'sirlashuvchi ikkita bir xil va har xil zarrachalar sistemasi gamiltonianlariga mos ikki zarrachali diskret Shredinger operatorlarining spektral xossalari Faria, Corolli, S.Albeverio, S.Laqayevlarning ishlarda o'rganilgan.

Ikki zarrachali diskret Shredinger operatori $h_\mu(0)$ bir zarrachali gamiltoniani h_μ ga unitar ekvivalent va shuning uchun bir zarrachali Shredinger operatorlarining spektral xossalarini o'rganish panjaradagi ko'p zarrachali Shredinger operatorlar spektral nazariyasida muhim o'rin tutadi.

MUHOKAMA VA NATIJALAR

Ushbu [S.Lakaev, I.Bozorov] ishda uch o'lchamli panjarada potensialli maydonda tashqi maydon ta'sirida kontakt va bir qadamda ta'sirlashuvga mos bir zarrachali gamiltonian qaralgan. Unga mos diskret Shredinger operatori xos qiymatlar sonining zarrachaning ta'sir energiyalari $\mu > 0$ va $\lambda > 0$ larga bog'liqligi to'la o'rganilgan.

B.Saymon va M.Klausning ishida uzluksiz Shredinger operatori xos qiymatlari uchun kichik $\lambda > 0$ larda $d = 1$ da yaqinlashuvchi qator, $d = 2$ da esa kichik $\lambda > 0$ larda asimptotik yoyilma olingan.

2. Bir zarrachali Shredinger operatorining spektri

$\mathbb{T} = (-\pi, \pi]$ bir o`lchamli tor. $L_2(\mathbb{T})$ kvadrati bilan integrallanuvchi funksiyalar Gilbert fazosi. $L^t{}_2(\mathbb{T}) \subset L_2(\mathbb{T})$ toq funksiyalar qism fazosi bo`lsin.

$L^t{}_2(\mathbb{T})$ da aniqlangan quyidagi operatorni qaraymiz:

$$h_\mu = h_0 + \mu v,$$

Bu yerda h_0 operator $\varepsilon(t)$ funksiyaga ko`paytirish operatori, ya'ni

$$(h_0 f)(t) = \varepsilon(t) f(t);$$
$$\varepsilon(t) = 1 - cos2t;$$

ta`sir operatori (qo`zg`atish operatori) $v$ operator esa integral operator bo`lib, $L^t{}_2(\mathbb{T})$da quyidagicha aniqlangan:

$$(vf)(t) = \frac{1}{2\pi} sin2t \int_{\mathbb{T}} sin2q \cdot f(q) dq;$$

$\mu > 0$ −zarrachaning tashqi muhit bilan ta'sir energiyasi.

Lemma 1. *Yuqorida aniqlangan h_μ operator $L^t{}_2(\mathbb{T})$ da o`z-o`ziga qo`shma, chegaralangan operator bo`ladi.*

Isbot. Hilbert fazosida o`z-o`ziga qo`shma operator ta`rifiga ko`ra $A: H \to H$ operator $\forall f, g \in H$ uchun $(Af, g) = (f, Ag)$ shartni qanoatlantirishi kerak. $(\cdot,\cdot)$ — skalyar ko`paytma.

O`z-o`ziga qo`shma operator xossasiga ko`ra h_0 va v operatorlarning har birining o`z-o`ziga qo`shma operator ekanligini ko`rsatish yetarli.

$$(h_0 f, g) = \int_{\mathbb{T}} (1 - cos2q) f(q) \overline{g(q)} dq == \int_{\mathbb{T}} f(q)(1 - cos2q)\overline{g(q)} dq =$$

$$= \int_{\mathbb{T}} f(q) \overline{(1 - cos2q) g(q)} dq = (f, h_0 g);$$

$$(vf, g) = \int_{\mathbb{T}} (\frac{1}{2\pi} sin2q \int_{\mathbb{T}} sin2t f(t) dt) \cdot \overline{g(q)} dq =$$

$$= \int_{\mathbb{T}} f(t) \overline{(\frac{1}{2\pi} sin2t \int_{\mathbb{T}} sin2q \cdot g(q)\, dq)} dt = (f, vg).$$

Chegaralanganligi. Operator chegaralanganlik ta`rifidan $A$ operator uchun $\exists M > 0$ soni mavjud bo`lib $\forall f \in D(A) \subset H$ element uchun quyidagi

$$\|(Af)\| \leq M\|f\|;$$

tengsizlik o`rinli bo`lsa, A operator chegaralangan deyiladi.

$$\|(h_\mu f)(t)\|^2 = \int_{\mathbb{T}} \left|(1 - cos2t) f(t) + \frac{\mu}{2\pi} sin2t \int_{\mathbb{T}} sin2q f(q) dq\right|^2 dt \leq$$

$$\leq 2 \int_{\mathbb{T}} |(1 - cos2t) f(t)|^2 dt + + 2 \int_{\mathbb{T}} \left|\frac{\mu}{2\pi} sin2t \int_{\mathbb{T}} sin2q f(q) dq\right|^2 dt$$

$$\leq 8 \int_{\mathbb{T}} |f(t)|^2 dt + 2 \int_{\mathbb{T}} \left|\frac{\mu}{2\pi} \int_{\mathbb{T}} sin2q f(q) dq\right|^2 dt =$$

$$= 8 \int_{\mathbb{T}} |f(t)|^2 dt + \frac{\mu^2}{\pi} \left|\int_{\mathbb{T}} sin2q f(q) dq\right|^2;$$

Koshi – Bunyakovskiy tengsizligiga ko`ra:

$$\left|\int_{\mathbb{T}} sin2q f(q) dq\right|^2 \leq \int_{\mathbb{T}} sin^2 2q dq \cdot \int_{\mathbb{T}} |f(q)|^2 dq$$

$$= \frac{1}{2} \int_{\mathbb{T}} (1 - cos2q) dq \cdot \int_{\mathbb{T}} |f(q)|^2 dq = \pi \|f\|^2;$$

Demak, $\|(h_\mu f)(t)\| \leq \sqrt{8 + \pi}\|f\|$. Lemma isbotlandi.▲

$v-$ chekli o`lchamli (bir o`lchamli) operator bo`lganidan uning kompakt ekanligiga kelamiz.

h_μ operator o`z-o`ziga qo`shma bo`lganligi uchun uning qoldiq spektri bo`sh to`plam bo`lib, spektri haqiqiy sonlar to`plamining qismidan iborat, ya'ni

$$\sigma(h_\mu) \subset \mathbb{R}.$$

Quyidagi belgilashlarni kiritamiz:

$$\varepsilon_{min} = \min_{q\in\mathbb{T}} \varepsilon(q) = 0; \ \varepsilon_{max} = \max_{q\in\mathbb{T}} \varepsilon(q) = 2;$$

Muhim spektr turg'unligi haqidagi Veyl teoremasiga ko'ra. h_μ operatorning muhim spektri $\sigma_{ess}(h_\mu)$ berilgan $\mu \geq 0$ parametrlardan bog'liq emas va h_0 operatorning spektri bilan ustma-ust tushadi. Shunday qilib

$$\sigma_{ess}(h_\mu) = \sigma_{ess}(h_0) = \sigma(h_0) = [\varepsilon_{min}, \varepsilon_{max}] = [0,2]$$

tenglik o'rinli.

3. Asosiy natijalar bayoni.

Teorema 1; $\mu > 1$ *bo`lsin* $h_\mu$ *muhim spektrdan o`ngda yagona xos qiymatga ega va u quyidagi ko`rinishga ega*

$$z(\mu) = 2 + \frac{(\mu-1)^2}{2\mu};$$

Bu xos qiymatga mos xos funksiyaning ko`rinishi.

$$f(q) = -\frac{2\mu^2 C sin2q}{\mu^2 + 2\mu cos2q + 1}$$

dan iborat.

Natija. *Quyidagi assimptotik formulalar o`rinli*

$$z(\mu) = 2 + \frac{1}{6}(\mu-1)^2 + o(\mu-1)^2$$

$$z(\mu) = \frac{\mu}{2} + O\left(\frac{1}{\mu}\right).$$

4. $\Delta_\mu(z)$ funksiyaning xossalari.

C orqali kompleks sonlar tekisligini belgilaymiz, har bir $\mu>0$ va $z \in C \setminus [0,2]$ da aniqlangan

$$\Delta_\mu(z) = 1 + \frac{\mu}{2\pi}\int_{\mathbb{T}} \frac{sin^2 2q}{\varepsilon(q) - z} dq; \quad (1)$$

funksiyani aniqlaymiz.

Bitiruv malakaviy ishini natijasini isbotlashda muhim o'rin tutadigan quyidagi lemmalarni isbotlaymiz.

Lemma 2. *$z \in C\backslash[0,2]$ soni h_μ operatorning xos qiymati bo'lishi uchun*

$$\Delta_\mu (z) = 0.$$

tenglikning bajarilishi zarur va yetarli.

Isbot; Zarurligi. $z \in C\backslash[0,2]$ soni h_μ operatorning xos qiymati bo'lisin. U holda

$$(\varepsilon(q) - z)f(q) = -\frac{\mu}{2\pi} sin2q \int_{\mathbb{T}} sin2tf(t)dt \quad (2)$$

tenglama nolmas $0 \neq f(q) \in L^t{}_2(\mathbb{T})$ yechimga ega.

Agar

$$\int_{\mathbb{T}} sin2tf(t)dt = C \quad (3)$$

deb belgilash kiritsak. ($C \neq 0$)

$$(\varepsilon(q) - z)f(q) = -\frac{\mu}{2\pi} sin2qC;$$

tenglikka kelamiz. Bundan $f(q)$ ni topamiz

$$f(q) = -\frac{\mu C}{2\pi} \frac{sin2q}{(\varepsilon(q) - z)}; \quad (4)$$

Ushbu (4) tenglik bilan aniqlangan $f(q)$ funksiyani (3) belgilashga keltirib qo`yamiz va quyidagi

$$C = -\frac{\mu C}{2\pi} \int_{\mathbb{T}} \frac{sin^2 2q}{(\varepsilon(q) - z)} dq$$

tenglikka ega bo`lamiz, ya`ni:

$$C + \frac{\mu C}{2\pi} \int_{\mathbb{T}} \frac{sin^2 2q}{(\varepsilon(q) - z)} dq = 0.$$

(1) belgilashga ko`ra

$$C\ \Delta_\mu (z) = 0.$$

$C \neq 0$ ekanligidan

$$\Delta_\mu (z) = 0$$

tenglikka kelamiz.

Yetarliligi. $\Delta_\mu (z) = 0$ bo`lsin. U holda $z \in C \setminus [0,2]$ soni $h_\mu$ operatorning xos qiymati bo`lishini ko`rsatamiz.

$$f(q) = -\frac{\mu C}{2\pi}\frac{sin2q}{(\varepsilon(q)-z)} = -\frac{\mu C}{2\pi}\frac{sin2q}{(1-cos2q-z)};$$

funksiyani qurib olamiz. Dastlab $f(q) \in L_2^t(\mathbb{T})$ bo`lishini ko`rsatamiz.

$$\int_{\mathbb{T}} \left|-\frac{\mu C}{2\pi}\frac{sin2q}{(1-cos2q-z)}\right|^2 dq = \left(\frac{\mu C}{2\pi}\right)^2 \int_{\mathbb{T}} \left(\frac{sin2q}{(1-cos2q-z)}\right)^2 dq;$$

$z \in C \setminus [0,2]$ ekanligidan $1-cos2q-z \neq 0$ va $\frac{sin2q}{(1-cos2q-z)}-$ uzluksiz funksiya. Bundan uning chegaralangan ekanligiga kelamiz. Demak, $f(q) \in L_2(\mathbb{T})$.

$sin2q-$ toq funksiya, $(1-cos2q-z)-$juft funksiya. Bundan $f(q)-$ toq funksiya. Demak, $f(q) \in L_2^t(\mathbb{T})$.

Endi bu funksiya z soni uchun (1) tenglamaning yechimi bo`lishini ko`rsatamiz.

$$(\varepsilon(q)-z)\left(-\frac{\mu C}{2\pi}\frac{sin2q}{(1-cos2q-z)}\right)$$
$$= -\frac{\mu}{2\pi}sin2q\int_{\mathbb{T}} sin2t\left(-\frac{\mu C}{2\pi}\frac{sin2t}{(1-cos2t-z)}\right)dt.$$

Ushbu tenglikdan

$$sin2q + \frac{\mu}{2\pi}sin2q\int_{\mathbb{T}} \frac{sin^2 2t}{(1-cos2t-z)}dt = 0$$

ga kelamiz, ya`ni:

$$sin2q\left(1+\frac{\mu}{2\pi}\int_{\mathbb{T}} \frac{sin^2 2t}{(1-cos2t-z)}dt\right) = 0;$$

(1) belgilashga ko`ra

$$\Delta_\mu(z)sin2q = 0;$$
$$\Delta_\mu(z) = 0.$$

bundan (2) tenglikning o`rinli ekanligiga kelamiz.

Lemma isbot bo`ldi.

Lemma 2.3. *$\Delta_\mu(z)$ funksiya $z \in (2;\infty)$da monoton o`suvchi bo`lib*

$$\lim_{z\to\infty}\Delta_\mu(z) = 1;$$
$$\lim_{z\to 2}\Delta_\mu(z) = 1-\mu.$$

Isbot; (1) formula bilan aniqlangan $\Delta_\mu(z)$ funksiyadan z bo`yicha hosila olamiz.

$$\frac{\partial \Delta_{\mu}(z)}{\partial z} = \mu \int_{\mathbb{T}} \frac{sin^2 2q}{(\varepsilon(q) - z)^2} dq > 0.$$

Demak, $\Delta_{\mu}(z)$ funksiya monoton o`suvchi.

$$\frac{sin^2 2q}{\varepsilon(q) - z}$$

funksiya $z > 2$ da uzluksiz va

$$\lim_{z\to\infty} \frac{sin^2 2q}{\varepsilon(q) - z} = 0;$$

ekanligidan, integral belgisi ostida limitga o`tish haqidagi teoremaga ko`ra:

$$\lim_{z\to\infty} \Delta_{\mu}(z) = 1;$$

ga kelamiz.

$sin^2 x + cos^2 x = 1$ ayniyatga ko`ra:

$$\lim_{z\to 2}\Delta_{\mu}(z) = \lim_{z\to 2}\left(1 + \frac{\mu}{2\pi}\int_{\mathbb{T}} \frac{sin^2 2q}{\varepsilon(q) - z} dq\right) == 1 + \frac{\mu}{2\pi}\lim_{z\to 2}\int_{\mathbb{T}} \frac{sin^2 2q}{\varepsilon(q) - z} dq =$$
$$= 1 + \frac{\mu}{2\pi}\int_{\mathbb{T}} \lim_{z\to 2}\frac{sin^2 2q}{\varepsilon(q) - z} dq == 1 - \frac{\mu}{2\pi}\int_{\mathbb{T}} \frac{sin^2 2q}{1 + cos2q} dq =$$
$$= 1 - \frac{\mu}{2\pi}\int_{\mathbb{T}} \frac{1 - cos^2 2q}{1 + cos2q} dq = = 1 - \frac{\mu}{2\pi}\int_{\mathbb{T}} (1 - cos2q) dq = 1 - \mu.$$

Lemma isbotlandi. ▲

Yuqoridagi lemmalardan kelib chiqadiki. $0 < \mu < 1$ bo`lsa

$$\Delta_{\mu}(z) = 0$$

tenglama yechimga ega emas. $\mu > 1$ da faqat bitta yechimga ega bo`ladi.

$\mu = 1$ bo`lsin.

$\Delta_{\mu}(z) = 0;$

tenglama $z = 2$ yechimga ega. $z = 2 \in \sigma_{ess}(h_{\mu})$. Bu son h_{μ} operatorning xos qiymat bo`lishini tekshiramiz.

$$(h_{\mu} f)(t) = 2f(t);$$

tenglamani qaraymiz. Bu tenglamadan

$$(1 + cos2t)f(t) = \frac{1}{2\pi} sin2t \int_{\mathbb{T}} sin2q f(q)\, dq;$$

ga kelamiz.

$$\int_{\mathbb{T}} sin2qf(q)\, dq = C;$$

belgilash olamiz ($C \neq 0$). Bundan

$$f(t) = \frac{C}{2\pi} \cdot \frac{sin2t}{1 + cos2t};$$

$f(t) \neq 0$, endi $f(t) \in L_2^t(\mathbb{T})$ ni tekshiramiz.

$$\frac{4\pi^2}{C} \int_{\mathbb{T}} f^2(t)dt = \int_{\mathbb{T}} \frac{sin^2 2t}{(1 + cos2t)^2} dt = \int_{\mathbb{T}} \frac{1 - cos2t}{1 + cos2t} dt$$

$$= \int_{-\pi}^{0} \frac{1 - cos2t}{1 + cos2t} dt + \int_{0}^{\pi} \frac{1 - cos2t}{1 + cos2t} dt = = 2 \int_{0}^{\pi} \frac{1 - cos2t}{1 + cos2t} dt \geq$$

$$\left(bu\ xosmas\ integral\ bo`lib\ t = \frac{\pi}{2}\ da\ maxsuslikka\ ega.\right)$$

$$\geq 2 \int_{\frac{\pi}{2}-\delta}^{\frac{\pi}{2}+\delta} \frac{1 - cos2t}{1 + cos2t} dt \geq C \int_{\frac{\pi}{2}-\delta}^{\frac{\pi}{2}+\delta} \frac{dt}{\left(t - \frac{\pi}{2}\right)^2} = C \int_{-\delta}^{\delta} \frac{dy}{y^2} = 2C_1 \int_{0}^{\delta} \frac{dy}{y^2}.$$

Ushbu munosabatning o`ng qismida turgan integral uzoqlashuvchi. Demak $f(t) \notin L_2^t(\mathbb{T})$. $z = 2$ xos qiymat emas.

Asosiy natijalar isboti

2.1Teoremaning isboti.

Dastlab,

$$\int_{\mathbb{T}} \frac{sin^2 2q}{\varepsilon(q) - z} dq$$

integralni hisoblaymiz .

$$\int_{\mathbb{T}} \frac{sin^2 2q}{\varepsilon(q) - z} dq = \int_{\mathbb{T}} \frac{sin^2 2q}{1 - cos2q - z} dq = \int_{\mathbb{T}} \frac{sin^2 2q}{A - cos2q} dq;$$

bu yerda

$$A = 1 - z < -1;$$

Trigonometriyaning asosiy ayniyatidan foydalanib integralni ikki qismga ajratamiz.

$$\int_{\mathbb{T}} \frac{sin^2 2q}{A - cos2q} dq = \int_{\mathbb{T}} \frac{1 - cos^2 2q}{A - cos2q} dq = \int_{\mathbb{T}} \frac{dq}{A - cos2q} - - \int_{\mathbb{T}} \frac{cos^2 2q}{A - cos2q} dq$$
$$= I_1 + I_2; \ (5)$$

Birinchi I_1 integralni hisoblaymiz:

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - cos2q};$$

Juft funksiyadan simmetrik oraliq bo`yicha olingan integral xossasidan foydalanib quyidagi tenglikka kelamiz:

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - cos2q} = \int_{-\pi}^{0} \frac{dq}{A - cos2q} + \int_{0}^{\pi} \frac{dq}{A - cos2q} = = 2\int_{0}^{\pi} \frac{dq}{A - cos2q}.$$

Quyidagicha almashtirish olamiz

$$e^{2iq} = \xi;$$

bundan quyidagilar hosil bo`ladi:

$$cos2q = \frac{1}{2}\left(\xi + \frac{1}{\xi}\right),$$
$$d\xi = 2ie^{2iq} dq;$$
$$dq = \frac{d\xi}{2i\xi};$$

Topilganlarni integralga qo`yamiz.

$$I_1 = 2\int_{0}^{\pi} \frac{dq}{A - cos2q} = 2\int_{|\xi|=1} \frac{d\xi}{2i\xi(A - \frac{1}{2}\left(\xi + \frac{1}{\xi}\right))} == \frac{2}{i}\int_{|\xi|=1} \frac{d\xi}{2A\xi - \xi^2 - 1}$$
$$= -\frac{2}{i}\int_{|\xi|=1} \frac{d\xi}{\xi^2 - 2A\xi + 1} = = -\frac{2}{i}\int_{|\xi|=1} \frac{d\xi}{(\xi - \xi_1)(\xi - \xi_2)};$$

bu yerda

$$\xi_1 = A - \sqrt{A^2 - 1};$$
$$\xi_2 = A + \sqrt{A^2 - 1};$$

Ko`rish mumkinki

$$|\xi_1| > 1, |\xi_2| < 1.$$

Koshining integral formulasi yordamida integralni hisoblaymiz.

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - cos2q} == -\frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{(\xi - \xi_1)(\xi - \xi_2)} = -\frac{2}{i} \cdot 2\pi i \cdot \frac{1}{\xi_2 - \xi_1}$$
$$= -\frac{2\pi}{\sqrt{A^2 - 1}};$$

Demak,

$$I_1 = -\frac{2\pi}{\sqrt{A^2 - 1}}. \qquad (6)$$

Endi I_2 integralni hisoblaymiz.

$$I_2 = -\int_{\mathbb{T}} \frac{cos^2 2q}{A - cos2q} dq; \qquad (7)$$

Integral ostidagi kasr ifoda suratiga A^2 ni qo`shib ayiramiz va integralni quyidagi ikkita qismga ajratamiz.

$$I_2 = -\int_{\mathbb{T}} \frac{cos^2 2q}{A - cos2q} dq = \int_{\mathbb{T}} \frac{A^2 - cos^2 2q - A^2}{A - cos2q} dq$$
$$= \int_{\mathbb{T}} (A + cos2q)dq - A^2 \int_{\mathbb{T}} \frac{dq}{A - cos2q} == 2\pi A - A^2 I_1;$$

Demak,

$$I_2 = -\int_{\mathbb{T}} \frac{cos^2 2q}{A - cos2q} dq = 2\pi A + \frac{2\pi A^2}{\sqrt{A^2 - 1}};$$

Lemma 2.4. ga ko`ra $z \in C \setminus [0,2]$ soni $h_\mu$ operatorning xos qiymati bo`lishi uchun $\Delta_\mu (z) = 0$ tenglamani qanoatlantirishi zarur va yetarli. Faraz qilamiz $z \in C \setminus [0,2]$ xos qiymat bo`lsin. U holda

$$\Delta_\mu (z) = 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{sin^2 2q}{\varepsilon(q) - z} dq = 0;$$

ga ega bo`lamiz. (5), (6) va (7) tengliklarga ko`ra.

$$1 + \frac{\mu}{2\pi}\left(-\frac{2\pi}{\sqrt{A^2 - 1}} + 2\pi A + \frac{2\pi A^2}{\sqrt{A^2 - 1}}\right) = 0;$$

Ushbu tenglik

$$1 + \mu\left(A + \frac{A^2 - 1}{\sqrt{A^2 - 1}}\right) = 0$$

ga ekvivalent. Bundan

$$1+\mu(A+\sqrt{A^2-1})=0;$$
$$\mu\sqrt{A^2-1}=-1-\mu A;$$
$$(A<-1,\mu>1\ bo`lganidan\ -1-\mu A>0)$$

Tenglamaning ikkala tomonini kvadratga ko`taramiz.

$$\mu^2A^2-\mu^2=1+2\mu A+\mu^2A^2.$$

O`xshash hadlarni qisqartirib soddalashtiramiz, natijada quyidagi A noma'lumga nisbatan chiziqli tenglamani hosil qilamiz:

$$1+2\mu A+\mu^2=0;$$

$A=1-z$ ekanligidan, quyidagi tenglikka kelamiz:

$$2\mu z=1+2\mu+\mu^2;$$
$$z=2+\frac{1-2\mu+\mu^2}{2\mu};$$

yoki

$$z=2+\frac{(1-\mu)^2}{2\mu}. \quad (8)$$

Bu xos qiymatga mos xos funksiyani aniqlaymiz. (3)ga ko`ra

$$f(q)=\frac{\mu C sin2t}{(1-cos2q-z)}$$

Demak, (8) ni hisobga olsak bu xos qiymatga mos xos funksiya:

$$f(q)=-\frac{2\mu^2 C sin2q}{\mu^2+2\mu cos2q+1}.$$

Ko`rinishda bo`lib bunda $C=const\neq 0$.

Teorema isbotlandi. ▲

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