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M U N D A R I J A

G’OVAK MUHITDA IKKI FAZALI SUYUQLIK SIZISHIDA QO’ZG’ALUVCHI CHEGARANI TOPISH MASALASINI SONLI ECHISH

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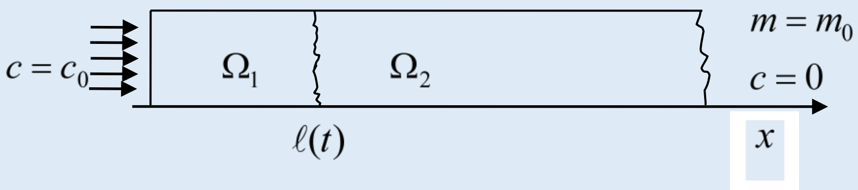
Yarim cheksiz bir o‘lchamli g‘ovak muhitni qaraylik. G‘ovak muhit toza suv bilan to‘ldirilgan uning g‘ovakligi m_0 ga teng. $t > 0$ vaqt momentida modda konsentratsiyasi c_0 bo‘lgan aralashma g’ovak muhitda harakatlana boshladi. (rasmga qarang)



1-Rasm. Bir jinsli suyuqlikning bir jinsli bo‘lmagan suyuqlikni haydash sxemasi

G‘ovak muhitda ikkita soha hosil bo‘ladi $\Omega_1 = (0, \ell)$ va $\Omega_2 = (\ell, \infty)$. Shu bilan birgalikda bu ikki soha orasida qo‘zg‘aluvchang chegara $\ell(t)$ ham mavjud bo‘ladi. Kolmatatsiya, suffoziya va adsorbsiya effektlari e’tiborga olinmaydi deb qaraymiz, ya’ni $m = m_0 = const$

$x = 0$ chegarada o‘zgarmas p_c bosimli oqim Ω_1 sohaga kirib kelmoqda. Bosimning boshlang‘ich taqsimoti $p(0, x) = p_0$ va u cheksiz $x = \infty$ gacha saqlanib qoladi.

Aralashmaning qovushqoqligi Eynshtey formulasi bilan o‘zgarsin $\mu_1 = \mu_0(1 + 2{,}5c_0)$, $x \in \Omega_1$, Ω_2 sohada esa $\mu_2 = \mu_0$, bu erda μ_0- toza suvning qovushqoqligi.

Muhitning o‘tkazuvchangligi bosimdan o‘zgarmaydi deb qaraymiz, ya’ni $k = const$.

Bosim maydoni uchun tenglama qo‘yidagi ko‘rinishda bo‘ladi [1,2,3]

$$\frac{\partial p_i}{\partial t} = \chi_i \frac{\partial^2 p_i}{\partial x^2} \tag{1}$$

p_i- Ω_i, $i = 1{,}2$ sohadagi bosim,

$$\chi_1 = \frac{\kappa}{\mu_1 \left(m_0 \beta_{ж} + \beta_c \right)},$$

$$\chi_2 = \frac{\kappa}{\mu_2 \left(m_0 \beta_{ж} + \beta_c \right)},$$

χ_i - Ω_i sohaning pezotkazuvchangligi

Masala shartidan (1) tenglama quyidagi shartlarda echiladi

$$p_1(0,x) = p_2(0,x) = p_0 \tag{2}$$

$$p_1(t,0) = p_c,\ \ p_2(t,\infty) = p_0 \tag{3}$$

$$p_1(t,\ell) = p_2(t,\ell) \tag{4}$$

$$\frac{\kappa}{\mu_1} \frac{\partial p_1(t,\ell)}{\partial x} = \frac{\kappa}{\mu_2} \frac{\partial p_2(t,\ell)}{\partial x} \tag{5}$$

$$m_0 \frac{d\ell}{dt} = -\frac{\kappa}{\mu_1} \frac{\partial p_1}{\partial x}\ . \tag{6}$$

Bu qo‘yilgan (1)-(6) masala odatda N.N.Verigin [3] masalasiga analogik jihatdan o‘xshashdir.

Bu masalani sonli usullarda yechamiz. (1) - (6) masalaning echimini sonli echish uchun to‘r kiritib, frontni ilib olish metodi (variable domain methods) dan foydalanamiz [4,5]. $\Omega = \Omega_1 \cup \Omega_2 = \{ 0 \le x \le L \}$ sohaga bir o‘lchamli to‘rni kiritamiz

$$\varpi_h = \varpi_I \cup \varpi_{II} = \{ x \mid x = x_i = ih, i = 0,1,..N, Nh = l \}$$

bu erda l – qatlamning xarakter uzunligi. Bu xarakter uzunlik bosimning etib borigan chegarasigacha olinadi, ya’ni qo‘zg‘aluvchang chegaragacha bo‘lgan qatlam sohasining uzunligi. Vaqt t bo‘yicha notekis to‘rni kiritamiz $\varpi_\tau = \{ t \mid t = t_j + \tau_j, t_0 = 0,1,..N, t_N = T \}$, bu erda $\tau_j > 0$ o‘zgaruvchang qiymat. Vaqt bo‘yicha qadam $\tau_{j+1}, j = 0,1,..N-1$ ko‘rinishda tanlab olinadi. Bunda vaqt intervalida (t_j dan t_{j+1} gacha) qo‘zg‘aluvchang chegarada fazo to‘ri bitta qadam siljiganda unga mos u ham bitta qadam siljiydi.

Chegaraviy to‘r dinamik to‘r deb ataladi va u $x = l(t_j) = i_j h$ ko‘rinishda aniqlanadi. (1) tenglamani oshkor bo‘lmagan ayirmali sxemada approksimatsiyalaymiz:

$$\frac{p_{1,i}^{j+1} - p_{1,i}^{j}}{\tau_{j+1}} = \chi_1 \frac{p_{1,i-1}^{j+1} - 2p_{1,i}^{j+1} + p_{1,i+1}^{j+1}}{h^2}, \qquad i = \overline{1, j}\ , \tag{7}$$

$$\frac{p_{2,i}^{j+1} - p_{2,i}^{j}}{\tau_{j+1}} = \chi_2 \frac{p_{2,i-1}^{j+1} - 2p_{2,i}^{j+1} + p_{2,i+1}^{j+1}}{h^2}, \quad i = \overline{j+2, N-1}. \tag{8}$$

Boshlang‘ich shart (2) ning approksimatsiyasi quyidagicha bo‘ladi

$$p_{1,i}^{0} = p_{2,i}^{0} = p_0, \ \ i = \overline{0, N}\,. \tag{9}$$

Chegaraviy shartlar (3), (4) va (5) quyidagi ko‘rinishga keladi

$$p_{1,0}^{j} = p_c, \ \ p_{2,N}^{0} = p_0, \tag{10}$$

$$i = j+1 \ \text{ tugundagi } \ p_{1,i+1}^{j+1} = p_{2,i+1}^{j+1}, \tag{11}$$

$$i = j+1 \text{ tugunda } \frac{\kappa}{\mu_1} \frac{p_{1,i}^{j+1} - p_{1,i-1}^{j+1}}{h} = \frac{\kappa}{\mu_2} \frac{p_{2,i+1}^{j+1} - p_{2,i}^{j+1}}{h}\ . \tag{12}$$

Endi (3.6) shartni opproksimatsiyalaymiz. (3.6) ni quyidagicha yozib olamiz:

$$\frac{\partial p_1}{\partial x} = -\lambda \frac{dl(t)}{dt}, \ \ x = l(t), \tag{13}$$

Bu erda $\lambda = \dfrac{m_0 \mu_1}{\kappa}$.

Qo’zg’aluvchang chegara bir qadam siljiganda quyidagicha aniqlanadi:

$$\frac{dl}{dt} \approx \frac{h}{\tau_{j+1}}.$$

(6) chegaraviy shartni opproksimatsiyalaganda h ning ikkinchi tartibini saqlab qolgan holda quyidagicha yozishimiz mumkin:

$$i = j+1 \text{ tugunda } \frac{\lambda h + 0{,}5\left(p_{1,i}^{j+1} - p_{1,i}^{j}\right)}{\tau_{j+1}} = \frac{p_{1,i}^{j+1} - p_{1,i-1}^{j+1}}{h}, \tag{14}$$

(7) va (8) tenglamalarni quyidagi ko'rinishga keltirib olibamiz

$$A_1 p_{1,i-1}^{j+1} - B_1 p_{1,i}^{j+1} + C_1 p_{1,i+1}^{j+1} = -F_1 \ , \ \ i = \overline{1, j} \ , \tag{15}$$

$$A_2 p_{2,i-1}^{j+1} - B_2 p_{1,i}^{j+1} + C_2 p_{2,i+1}^{j+1} = -F_2 \ , \ \ i = \overline{j+2, N-1}, \tag{16}$$

bu erda koeffisientlar quyidagiga teng

$$A_1 = \frac{\chi_1 \tau_{j+1}}{h^2} \ , \ B_1 = 1 + 2\frac{\chi_1 \tau_{j+1}}{h^2}, \ C_1 = \frac{\chi_1 \tau_{j+1}}{h^2}, \ F_1 = p_{1,i}^{j},$$

$$A_2 = \frac{\chi_2 \tau_{j+1}}{h^2} \ , \ B_2 = 1 + 2\frac{\chi_2 \tau_{j+1}}{h^2}, \ C_2 = \frac{\chi_2 \tau_{j+1}}{h^2}, \ F_2 = p_{2,i}^{j}.$$

(15)-(16), (14) chiziqlimas ayirmali masalami echish uchun har bir $t = t_{j+1}$ vaqt qatlamida iteratsiya jarayoni asosida yaqinlashib boradi. Iteratsiyon yaqinlashishni τ_{j+1} vaqt qadami deb olamiz. Agar boshlang'ich yaqinlashish τ_{j+1}^{0}deb olsak. Berilgan τ_{j+1}^{s} da mos ravishda $p_{1,i}^{j+1}$ va $p_{2,i}^{j+1}$ taqribiy echimlar uchun quyidagi chiziqli ayirmali masala echimiga kelamiz:

$$A_1 p_{1,i-1}^{s,j+1} - B_1 p_{1,i}^{s,j+1} + C_1 p_{1,i+1}^{s,j+1} = -F_1^{s} \ , \ \ i = \overline{1, j} \ , \tag{17}$$

$$A_2 p_{2,i-1}^{s,j+1} - B_2 p_{1,i}^{s,j+1} + C_2 p_{2,i+1}^{s,j+1} = -F_2^{s} \ , \ \ i = \overline{j+2, N-1}, \tag{18}$$

$$p_{2,\mathrm{N}}^{s,j+1} = p_0 , \tag{19}$$

$$p_{1,0}^{s,j+1} = p_c , \tag{20}$$

$$p_{1,\mathrm{i}+1}^{s,j+1} = p_{2,\mathrm{i}+1}^{s,j+1} , \tag{21}$$

$$\frac{\kappa}{\mu_1} \frac{p_{1,i}^{s,j+1} - p_{1,i-1}^{s,j+1}}{h} = \frac{\kappa}{\mu_2} \frac{p_{2,i+1}^{s,j+1} - p_{2,i}^{s,j+1}}{h} . \tag{22}$$

Bu tenglamalar sistemasi uch dioganalli pogonkaga keladi. (14) ayirmali munosabatdan vaqt qadamini aniqlab olishimiz mumkin

$$i = j+1 \text{ tugunda } \tau_{j+1}^{s+1} = \left(\lambda h + 0{,}5\left(p_{1,i}^{s,j+1} - p_{1,i}^{s,j}\right)\right)\left(\frac{p_{1,i}^{s,j+1} - p_{1,i-1}^{s,j+1}}{h}\right)^{-1} . \tag{23}$$

Endi (17)-(23) ayirmali masalani echish uchun qarama-qarshi progonka usulidan foydalanamiz, bu chap va o'ng progonka kombinatsiyasidan kelib chiqadi.

Bu erda $i = j+1,\ \ 0 < j+1 < N$ ichki tugunlar, qo'zg'aluvchang chegara shu to'r tugunlariga mos tushadi. Bu $0 \le i \le j$ sohada echimni o'ng progonka formulasidan foydalanib hisoblash mumkin:

$$p_{1,i}^{s,j+1} = \alpha_{i+1} p_{1,i+1}^{s,j+1} + \beta_{i+1},\ i = j, j-1, \ldots, 1, 0, \tag{24}$$

$$\alpha_{i+1} = \frac{B_1}{C_1 - A_1\alpha_i},\ i = 0, 1, \ldots, j,\ \alpha_1 = 0, \tag{25}$$

$$\beta_{i+1} = \frac{A_1\beta_i + F_1}{C_1 - A_1\alpha_i},\ i = 0, 1, \ldots, j,\ \beta_1 = p_c, \tag{26}$$

$j+1 \le i \le N$ sohada esa chap progonka formulasini qo'llaymiz:

$$p_{2,i+1}^{s,j+1} = \xi_{i+1} p_{2,i}^{s,j+1} + \eta_{i+1},\ i = j+1, j+2, \ldots, N, \tag{27}$$

$$\xi_i = \frac{A_2}{C_2 - B_2\xi_{i+1}},\ i = N-1, N-2, \ldots, j+1,\ \xi_N = 0, \tag{28}$$

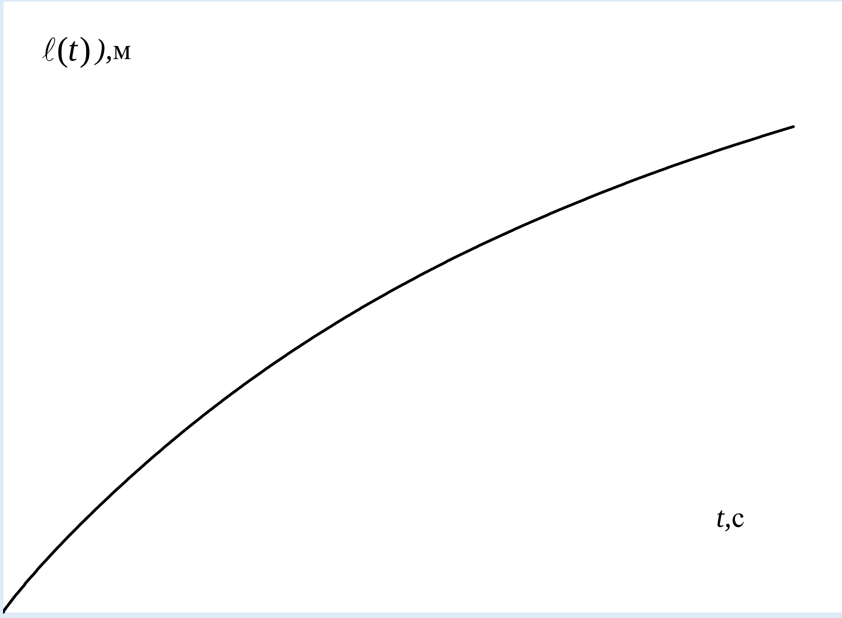
$$\mu_i = \frac{F_2 + B_2\eta_{i+1}}{C_2 - B_2\xi_{i+1}},\ i = N-1, N-2, \ldots, j+1,\ \eta_N = p_0, \tag{29}$$

(24), (27) larni (22) ga qo'ysak va $i = j+1$ da (21) ni hisobga olsak:

$$p_{1,j+1} = \frac{\mu_2\beta_{j+1} + \mu_1\eta_{j+2}}{\mu_2\left(1 - \alpha_{j+1}\right) + \mu_1\left(1 - \xi_{j+2}\right)} \tag{30}$$

ga ega bo'lamiz.

Sonli echim olish uchun suyuq sohaning o'tkazuvchangligi $\kappa = 10^{-12}$ m^2 va g'ovakligini $m_0 = 0{,}15$ deb olamiz. Itarayotgan va itarilayotga suyuqliklar bir jinsli va aralashmaning qovushqoqligi Enshtey formulasi bo'yicha $x \in \Omega_1$ sohada $\mu_1 = \mu_0\left(1 + 2{,}5c_0\right),\ \Omega_2$ sohada esa $\mu_2 = \mu_0,\ \mu_0 = 4 \cdot 10^{-4}$ Pa·s - $p_c - p_0 = 10$ bosim ostidagi qovushqoqlik. Bu berilganlarga mos $\ell(t)$ ning o'zgarishi 2-rasmda berilgan. Rasmdan ko'rish mumkibki, qo'zg'aluvchang chegara qatlamda monotong oshib borar ekan.



2-rasm. Qo‘zg‘aluvchang chegaraning harakati

FOYDALANILGAN ADABIYOTLAR

1. Федоткин И.М., Воробьев Е.И., Вьюн В.И. Гидродинамическая теория фильтрования суспензией. Киев:Вища. шк.,Головное изд-во.1986. -166с

2. Федоткин И.М. Математическое моделирование технологических процессов. Киев: Вища шк., Головные изд-во. 1988.-415с.

3. Нигматулин Р.И. Динамика многофазных сред. Т. II. М., Наука, 1987.-389с.

4. Самарский А.А., Вабищевич П.Н. Вычислительная теплопередача. –М.: Едиториал УРСС, 2003-784 с.

5. Самарский А.А. Теория разностных схем. – М.: Наука, 1977. -656с.