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M U N D A R I J A

O’QUVCHILARNING KREATIV QOBILIYATLARINI RIVOJLANTIRISHDA MANTIQ FANI ELEMENTLARIDAN FOYDALANISH

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Annotatsiya

Bu maqolada o’quvchilarga teoremalar isbotini va tengsizliklarni yechishga o’rgatishda predikatlar algebrasi formulalaridan foydalanishning ahamiyati ochib berilgan. Maqolada keltirilgan misollardan va teoremalardan o’quvchilarga nafaqat predikatlar algebrasining tadbiqlarini o’rgatishda, shuningdek kreativ qobiliyatli o’quvchilar bilan ishlashda va matematika fanidan to’garak mashg’ulotlarini tashkil qilishda ham foydalanish mumkin.

Kalit so’zlar: Kreativ qobiliyat, to’plam, dizyunksiya, konyunksiya, predikatlar, predikatning rostlik sohasi, tengkuchli formulalar, teorema isboti.

Matematika fani- matematik mantiq qonuniyatlari asosida o’rganiladi va rivojlantirib boriladi. Shunday bo’lsada matematik mantiq fani umumta’lim maktablarida alohida fan sifatida o’qitilmaydi. Matematika fani darsliklariga matematik mantiq fani elementlari qisman kiritilgan bo’lsada uning tadbiqlari yetarlicha yoritilmagan. Natijada o’quvchilar matematika fanining nazariy asoslarini chuqur o’rganishda, tenglama va tengsizliklarni yechishda, ayniqsa teoremalarni isbotlashda ko’p qiyinchiliklarga duch kelishmoqdalar. Shularni inobatga olib ushbu maqolada biz predikatlar algebrasining tengsizlik va tengsizliklar sistemasini yechishga hamda teoremalar isbotlashga ba’zi tadbiqlarini ko’rib chiqamiz.

Predikatlar algebrasining tadbiqlarini o’rganishda uning tengkuchli formulalarini bilish muhum hisoblanadi. Predikatlar algebrasining asosiy tengkuchli formulalarini yodga olamiz:

$$P(x)\wedge(S(x)\vee Q(x)) \equiv P(x)\wedge S(x)\vee P(x)\wedge Q(x) \quad (1)$$

$$P(x)\vee S(x)\wedge Q(x) \equiv (P(x)\vee S(x))\wedge(P(x)\vee Q(x)) \quad (2)$$

$$\overline{P(x)\wedge S(x)} \equiv \bar{P}(x)\vee\bar{S}(x) \quad (3)$$

$$\overline{P(x)\vee S(x)} \equiv \bar{P}(x)\wedge\bar{S}(x) \quad (4)$$

$$P(x) \Longrightarrow S(x) \equiv \bar{P}(x)\vee S(x) \quad (5)$$

$$P(x) \Longrightarrow S(x) \equiv \bar{S}(x) \Longrightarrow \bar{P}(x) \quad (6)$$

$$P(x) \Leftrightarrow S(x) \equiv P(x)\wedge S(x)\vee\bar{P}(x)\wedge\bar{S}(x) \quad (7)$$

$$P(x) \Leftrightarrow S(x) \equiv (\bar{P}(x)\vee S(x))\wedge(\bar{S}(x)\vee P(x)) \quad (8)$$

$$E_{\bar{p}} = \bar{E}_p. \quad (9)$$

$$E_{p\vee s} = E_p \cup E_s. \quad (10)$$

$$E_{p\wedge s} = E_p \cap E_s. \quad (11)$$

$$E_{p\Rightarrow s} = \overline{E_p} \cup E_s \quad (12)$$

$$E_{p\Leftrightarrow s} = (E_p \cap E_s) \cap (\bar{E}_p \cap \bar{E}_s) \quad (13)$$

$$E_{p\Leftrightarrow s} = (\bar{E}_p \cup E_s) \cap (\bar{E}_s \cup E_p). \quad (14)$$

Tengsizliklar predikatlardan iborat bo’lgani uchun tengsizlikni yechish masalasi predikatning rostlik sohasini topish masalasiga keladi. $P(x)$ va $S(x)$ lar biror $\mathcal{M}$ to’plamda aniqlangan predikatlar bo’lsin. Bu predikatlarning rostlik sohalarini mos ravishda E_p va E_s lar bilan, P(x) predikatning inkorini $\bar{P}(x)$ bilan va $\mathcal{M}\backslash E_p$ to’plamni $\bar{E}_p$ bilan belgilaymiz. $\bar{P}(x)$, $P(x)\vee S(x)$, $P(x)\wedge Q(x)$, $P(x) \Longrightarrow S(x)$ va $P(x) \Leftrightarrow S(x)$ predikatlarning rostlik sohalarini topishda (9)-(14) formulalardan foydalanamiz.

Bu formulalarning isboti predikatlar ustida amallarning ta’riflaridan va predikatlar algebrasining yuqorida keltirilgan (1)-(8) tengkuchli formulalaridan kelib chiqadi [4].

R - haqiqiy sonlar to’plami bo’lsin.

1-misol. R to’plamda $x^2-6x-7<0$ predikat berilgan. Uning rostlik sohasini toping.

Yechish. Berilgan predikatni $P(x)$ bilan, uning rostlik sohasini E_p bilan belgilab olamiz. U holda,

P(x) $\equiv$ (x2-6x-7<0) $\equiv$ (x+1)(x-7)<0) $\equiv$ (x+1<0)^(x-7>0)$\vee$(x+1>0)(x-7<0) $\equiv$

(x<-1)∧(x-7)∨(x>-1)∧(x<7) , [5] formuladan va (10), (11) dan topamiz,

E_p=(-∞ ;-1)∩(5: ∞) W (-1; ∞)X(-∞; 5) = :W(-1;5)=(-1;5) bo'lib umumiy yechim (-1;5) kelib chiqdi. Javob: E_p=(-1;5) .

2-misol. *R* to'plamda aniqlangan *P*(*x*) = (x^2-*x*-20>0) predikat berilgan. Uning rostlik sohasi E_p ni toping.

Yechish. $P(x) \equiv (x^2 - x - 20 > 0) \equiv ((x+4)\cdot(x-5) > 0) \equiv$

$\equiv (x+4<0) \wedge (x-5<0) \vee (x+4>0) \wedge (x-5>0) \equiv$

$\equiv (x<-4) \wedge (x<5) \vee (x>-4) \wedge (x>5)$, Bundan va (10), (11) dan topamiz, $E_p = \{x \in R | x < -4\} \cap \{x \in R | x < 5\} \cup$

$\cup \{x \in R | x > -4\} \cap \{x \in R | x > 5\} = (-\infty; -4) \cap (-\infty; 5) \cup$

$\cup (-4; \infty) \cap (5; \infty) = (-\infty; -4) \cup (5; \infty)$.

Javob: $E_p = (-\infty; -4) \cup (5; \infty)$.

3-misol. R to'plamda $P(x) = \left(\frac{2x+6}{5x-10} \leq 0\right)$ predikat berilgan. Uning rostlik sahasi E_p ni toping.

Yechish. $P(x) = \left(\frac{2x+6}{5x-10} \leq 0\right) \equiv (2x+6 \leq 0) \wedge (5x-10>0) \vee$

$\vee (2x+6 \geq 0) \wedge (5x-10<0) \equiv (x \leq -3) \wedge (x>2) \vee (x \geq -3) \wedge (x < 2)$.

Bundan va (10), (11) dan topamiz, $E_p = \{x \in R | x \leq -3\} \cap \{x \in R | x > 2\} \cup$
$\cup \{x \in R | x \geq -3\} \cap \{x \in R | x < 2\} = (-\infty; -3] \cap (2; \infty) \cup [-3; \infty) \cap$

$\cap (-\infty; 2) = \emptyset \cup [-3; 2) = [-3; 2)$. Javob: $E_p = [-3; 2)$.

4-misol. R to'plamida *P*(*x*)=(|*x*-2|<3) predikat berilgan. Uning rostlik sohasi E_p ni toping.

Yechish. $P(x) = (|x-2| < 3) \equiv (x-2<3) \wedge (x-2>-3) \equiv$

$\equiv (x<5) \wedge (x>-1)$. Bundan va (11) dan,

$E_p = \{x \in R | (x<5) \wedge (x>-1)\} = \{x \in R | x<5\} \cap \{x \in R | x > -1\} =$

$= (-\infty; 5) \cap (-1; \infty) = (-1; 5)$. Javob: E_p=(-1;5) .

5-misol. R to’plamda aniqlangan $P(x) = (|2x+6| \geq 4)$ predikat berilgan. Uning rostlik sohasi E_p ni toping.

Yechish. $P(x) = (|2x+6| \geq 4) \equiv (2x+6 \geq 4) \vee (2x+6 \leq -4) \equiv$

$\equiv (2x \geq -2) \vee (2x \leq -10) \equiv (x \geq -1) \vee (x \leq -5)$. Bundan va (11) dan,

$E_p = \{x \in R | (x \geq -1) \vee (x \leq -5)\} = \{x \in R | x \geq -1\} \cup$

$\{x \in R | x \leq -5\} =$

$= [-1; \infty) \cup (-\infty; -5] = (-\infty; -5] \cup [-1; \infty)$, kelib chiqadi.

Javob: $E_p = (-\infty; -5] \cup [-1; \infty)$.

6-misol. R to’plamda aniqlangan $P(x) = (x^2 - x \leq 0)$ va $S(x) = (x \leq \sqrt{x})$ predikatlar berilgan. E_p =?, E_s =?, $E_{p\wedge s}$ =?, $E_{p\vee s}$ =?, $E_{p\Rightarrow s}$ =?, $E_{s\Rightarrow p}$ =?,

$E_{p\Leftrightarrow s}$ =? topilsin.

Yechish. (9)-(14) formulalardan foydalanamiz.

$E_p = \{x \in R | x^2 - x \leq 0\} = \{x \in R | x(x-1) \leq 0\} = \{x \in R | (x \leq 0) \wedge$

$\wedge (x-1) \geq 0\} \vee (x - 1 \leq 0) \wedge (x \geq 0)\} = \{x \in R | x \leq 0\} \cap \{x \in R | x \geq$

$1\} \cup$

$\cup \{x \in R | x \leq 1\} \cap \{x \in R | x \geq 0\} = (-\infty; 0] \cap [1; \infty) \cup (-\infty; 1] \cap$

$[0; \infty) =$

$= \emptyset \cup [0; 1] = [0; 1]$; $E_p = [0; 1]$.

$E_s = \left\{x \in R \middle| x \leq \sqrt{x}\right\} = \{x \in R | (x \geq 0) \wedge (x^2 \leq x)\} = \{x \in R | x \geq 0\} \cap$

$\cap \{x \in R | x(x-1) \leq 0\} = [0; \infty) \cap [0; 1] = [0; 1]$. $E_s = [0; 1]$.

$E_{p\wedge s} = E_p \cap E_s = [0; 1] \cap [0; 1] = [0; 1]$

$E_{p\vee s} = E_p \cup E_s = [0; 1] \cup [0; 1] = [0; 1]$

$E_{p\Rightarrow s} = \bar{E}_p \cup E_s = (-\infty; 0) \cup (1; \infty) \cup [0; 1] = (-\infty; \infty)$.

Shunga o’xshash $E_{s\Rightarrow p} = (-\infty; \infty)$. $E_{p\Leftrightarrow s} = E_{p\Rightarrow s} \cap E_{s\Rightarrow p} = (-\infty; \infty)$.

O’quvchilarga predikatlar algebrasining tengkuchli formulalaridan foydalanib isbotlashga doir masalalar yechishni o’rgatishda quyidagi teoremalardan foydalanish mumkin.

1-teorema. $(\forall x \in R)(x^2 \leq x \Rightarrow x \leq \sqrt{x})$.

2-teorema. $(\forall x \in R)(x \leq \sqrt{x} \Longrightarrow x^2 \leq x)$.

3-teorema. $(\forall x \in R)(x \leq \sqrt{x} \Leftrightarrow x^2 \leq x)$.

4-teorema. $(\forall x \in \mathcal{M})(P(x) \Longrightarrow S(x)) \Longrightarrow (E_p \subset E_s)$.

5-teorema. $E_p \subset E_s \Longrightarrow (\forall x \in \mathcal{M})(P(x) \Longrightarrow S(x))$.

6-teorema. $(\forall x \in \mathcal{M})(P(x) \Leftrightarrow S(x)) \Longrightarrow (E_p = E_s)$.

7-teorema. $(E_p = E_s) \Longrightarrow (\forall x \in \mathcal{M})(P(x) \Leftrightarrow S(x))$.

Bu teoremalar teskarisidan faraz qilish usuli bilan oson isbotlanadi. Biz 7-teoremani isbotini keltirish bilan cheklanamiz.

Isbot. Teskarisidan faraz qilish usulidan foydalanamiz.

$$\overline{(\forall x \in \mathcal{M})\big(P(x) \Leftrightarrow S(x)\big)} \equiv (\exists x \in \mathcal{M})\overline{(P(x) \Leftrightarrow S(x))} \equiv$$

$$\equiv (\exists x \in \mathcal{M})\overline{(P(x) \wedge S(x) \vee \bar{P}(x) \wedge \bar{S}(x))} \equiv$$

$$\equiv (\exists x \in \mathcal{M})(\big(\bar{P}(x) \vee \bar{S}(x)\big) \wedge \big(P(x) \vee S(x)\big)) \equiv$$

$$\equiv (\exists x \in \mathcal{M})(\bar{P}(x) \wedge S(x) \vee \bar{S}(x) \wedge P(x)) \quad [5].$$

Bundan $(\exists x \in \mathcal{M})((\bar{P}(x) \wedge S(x) = 1) \vee (\bar{S}(x) \wedge P(x) = 1)) \equiv$ $(\exists x \in \mathcal{M})((\bar{P}(x) = 1) \wedge (S(x) = 1) \vee (\bar{S}(x) = 1) \wedge (P(x) = 1)) \equiv$

$$(\exists x \in \mathcal{M})((P(x) = 0) \wedge (S(x) = 1) \vee (S(x) = 0) \wedge (P(x) = 1)) \equiv$$

$$(\exists x \in \mathcal{M})((\overline{x \in E_p}) \wedge (x \in E_s) \vee (\overline{x \in E_s}) \wedge (x \in E_p)) \Longrightarrow$$

$\Longrightarrow \overline{E_p = E_s} \Longrightarrow E_p \neq E_s$. Teorema isbot bo'ldi.

Yuqorida ko'rib chiqilgan misol va masalalardan o'quvchilarga predikatlar algebrasining tadbiqlarini o'rgatishda foydalanish mumkin. O'quvchilarga matematik mantiq fani qonuniyatlari, keltirib chiqarish qoidalari, tengkuchli formulalari va ularning tadbiqlari chuqur va atroflicha o'rgatib borilsa, ularning muammoli vaziyatlarni tez va hatosiz hal qilish qobiliyatlari rivojlanib boradi.

Foydalanilgan adabiyotlar

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