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M U N D A R I J A

THE SPECTRAL PROPERTIES OF THE ONE-PARTICLE SCHODINGER OPERATOR ON THE TWO-DIMENSIONAL LATTICE

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Introduction

Some spectral properties of the Schrodinger operators, corresponding to energy operators-Hamiltonians of one and the system of two quantum particles moving on lattices, are studied by S.Albeverio, S.N.Lakaev, Z.Muminov, Faria da Veiga P.A.,Ioriatti L., and O'Carrol M/

The earliest result relating positivity and the nondegeneracy of an eigenvalue go back to afundamental theorem of Perron and Frobenius: a finite matrix with ssrictly positive elements always has its specral radius as an eigenvalue of multiplicity one with the corresponding eigenvector strictly positive. The Perron and then F.G.Frobenius.

The idea of applying a theorem of Perron-Frobenius type to quantum systems ie due to J.Glimm and A.Jaffe. The idea ofusing the irreducibility which simplifies the proof is due to I.Segal. The application to nonrelativistic systems is due to B.Simon and R.Hoegh-Krohn.

In the book of M.Reed and B.Simon the theorem of Perron-Frobenius type for the Hamiltonian of an N-body Schrodinger system with center of mass motion removed.

RESULT AND DISCUSSION

In the present paper we consider the Hamiltonian $h=h_0-V_{\mu\lambda}$,describing the energy of one qquantum particle on the two-dimensional lattice Z^2 and moving in the potential field $V_{\mu\lambda}$ are defined by

$$(h_0\psi)(x)=\Sigma_{s\epsilon Z^2}\tilde{\varepsilon}\,(s)\psi(x+s),\quad (V_{\mu\lambda}\psi)(x)=v_{\mu\lambda}(x)\psi(x),\ \ \psi\epsilon l_2(Z^2),$$

where we assume the functions $\tilde{\varepsilon}$ (s) and $v_{\mu\lambda}(x)$ to be defined by

$$\tilde{\varepsilon}(s)=\begin{cases} 2\,, & s=0 \\ -\frac{1}{2}\,, & s \in \{\pm e_1\,,\ \pm e_2\}, \\ 0\,, & otherwise \end{cases}$$

$$V_{\mu\lambda}(x)=\begin{cases} \mu\,, & x=0 \\ \lambda\,, & x=e_1 \\ 0\,, & otherwise \end{cases}$$

Here e_1, e_2 are the elements of the canonical basis of Z^2 and μ,λ are arbitrary positive numbers.

Theorem. For any $\forall\ \mu,\lambda > 0$, $\mu \neq \lambda$, ($\mu=\lambda$) the operator h has two simple eigenvalues (an eigenvalues of multiplicity two), lying below the bottom of the essential spectrum, with the (positive) eigenfunctions belong to $l_2(Z_0^2)$ and $l_2(Z_1^2)$, where Z), where Z_0^2 resp. Z_1^2 the subset Zx2Z resp. Zx(2Z+1) of two-dimensional lattice Z^2.

CONCLUSION

The main result of the present paper refers to the one-particle Schrodinger operator that has an eigenvalue as the lowest point in its spectrum . Under certain conditions, we show that the eigenspace corresponding to this eigenvalue may be one or two dimensional and that corresponding eigenvector is a positive function.

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